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Displacement-based nonlinear finite element analysis of composite beam–columns with partial interaction

João Batista M. Sousa Jr. *, Claudio E.M. Oliveira, Amilton R. da Silva

Dept. Civil Engnrg., Escola de Minas, Universidade Federal de Ouro Preto, 35400-000, Ouro Preto, MG, Brazil

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ABSTRACT

Several structures employ members with a deformable connection between two components, also known as partial interaction or interlayer slip. Classical examples include composite steel–concrete beams and glued or nailed timber beams. Most of the numerical research in this topic, however, has been focused on composite beams with material nonlinearity. In some situations these members may be subjected to compression and bending, for which 2nd order effects should be taken into account. The purpose of this work is to develop and test a displacement-based finite element model for the nonlinear material and geometrical analysis of composite beam–columns with interlayer slip. The finite element is based on a total Lagrangian description, in the context of large displacements, small strains and moderate rotations. The robustness and accuracy of the proposed scheme is verified against examples from the literature.

1. Introduction

It is common practice in civil engineering to employ members which consist of an assemblage of two or more different elements connected by a deformable interface. Examples are composite steel–concrete beams, where the connection is normally done by shear connectors welded to the steel beam and embedded in the concrete, and composite wood beams, glued or connected by nails or other devices.

Early works dealing with composite beams with interlayer slip focused on the development of closed-form solutions. The differential equations for the case of linear elastic composite beams with linear connection stiffness were originally developed in the early 50's [1]. Numerical methods have been employed to solve more general problems of composite beams, mostly by the finite element method. Displacement-based [2,3], force-based [4] and mixed [5,6] finite elements have been largely used in problems with linear and nonlinear material behaviour. A comparative study of different schemes is given by Ranzi et al. [7].

Partly because of the assumptions commonly used in the analysis of composite steel–concrete beams, few works have dealt with geometrical nonlinear analysis on beam–columns with interlayer slip, or even with the influence of the normal force without 2nd order effects. However, such situation is not uncommon, as elements with a deformable connection may eventually be part of a frame resisting vertical and lateral loads.

Nonlinear geometric effects were addressed by Girhammar and Gopu [8], who obtained the differential equations and boundary conditions for beam–columns with interlayer slip and linear material behaviour. Later, Girhammar and Pan [9] extended the work on these closed-form solutions, discussing the consideration of different boundary conditions and buckling lengths of beam–columns with interlayer slip. Xu and Wu [10] developed the analytical formulation for geometric nonlinear problems within the context of Timoshenko beam theory.

Numerical solutions for the geometric nonlinear problem of beams with deformable connection were investigated by Čas et al. [11]. These authors developed a nonlinear finite element scheme based on strain interpolation, under Reissner beam theory. Pi et al. [12] developed a total Lagrangian formulation for nonlinear geometrical and material analysis of composite beam columns and provided an extensive discussion on aspects such as steel and concrete plasticity, importance of shear strains and consistent linearisation of the virtual displacement principle. The relative slip between the steel and concrete components due to a flexible bond at the interface between the steel and concrete components was considered as an independent displacement. The implementation of a FE model and the respective incremental-iterative solution was described in a companion paper [13]. Krawczyk and coworkers [14,15] addressed the problem analytically and developed a corotational formulation for the nonlinear analysis of composite beams with interlayer slips, based on Timoshenko beam theory. Very recently, Battini et al. [16] developed the corotational transformations which can be employed to derive formulations for large displacements and

* Corresponding author. Tel.: +55 31 35591564; fax: +55 31 35591548.
E-mail addresses: joao@em.ufop.br, joaobatistasousajr@hotmail.com (J.B.M. Sousa Jr.), claudioernani@yahoo.com.br (C.E.M. Oliveira), amilton@metalicas.em.ufop.br (A.R. da Silva).

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rotations of beams with interlayer slips. Within the corotational framework different "local" elements may be employed. In their work, they exemplified the procedure applying it to a local element based on the exact solution and presented several examples.

There is still room for research, however, for cases of combined material and geometrical nonlinearities. The purpose of the present work is to develop a total Lagrangian displacement-based finite element formulation for the nonlinear material and geometrical analysis of composite beam–columns with interlayer slip, which is able to handle nonlinear effects such as those present in composite steel–concrete beam–columns. It is based on previously developed finite element formulations, such as the ones developed in [2], taking into account small strain and moderate rotation effects. The slip is obtained from the beam displacements and rotation, so there is no need for a separate slip interpolation. At the end, some examples of nonlinear material and geometrical composite beams are presented to illustrate the accuracy and robustness of the finite element formulation. The results are verified against analytical as well as numerical solutions.

2. Formulation

The kinematical hypotheses for the composite beam are based on the Euler–Bernoulli beam theory, namely, cross sections remain plane and normal to the reference axis after deformation. This gives rise to expressions involving trigonometric functions on the beam cross section rotations which are essential if one wishes to take into account the general finite deformation case. However, in civil engineering applications and especially with reinforced concrete, the usual framework is that of small strains and moderate rotations. Dall'Asta et al. [17] and Zona et al. [18] have discussed the mathematical aspects of these simplifications on the strain field for nonlinear geometrical and material analysis of prestressed beams, and equivalent assumptions are applied in the present work.

Within the simplified kinematical scheme, with superscript 0 denoting values measured at the reference axis, the in-plane beam displacements u_α and v are mathematically expressed as

$$u_\alpha(x,y) = u_\alpha^0(x) - (y - y_\alpha)\theta(x) = u_\alpha^0(x) - (y - y_\alpha)v_{,x}^0 \quad \alpha = 1,2 \tag{1}$$

$$v_\alpha(x,y) = v^0(x) \tag{2}$$

for the displacements of the components and

$$s(x) = u_2^0(x) - u_1^0(x) + h\, v_{,x}^0 \tag{3}$$

for the slip s between the components (Fig. 1), with $h = y_2 - y_1$. These equations have been employed in several formulations for composite beam analysis in the geometric linear regime. In the case of geometric nonlinearity, large displacements and moderate rotations also affect the kinematics of the interlayer slip, and the horizontal and vertical components of s should be evaluated separately. However, if it is assumed that the slips themselves are moderate (as the rotations), Eq. (3) remains valid for practical engineering applications and as such was employed by Girhammar and coworkers [8,9] in their analytical formulations.

Considering a total Lagrangian formulation, the only strains in the beams are the Green–Lagrange axial strains, which in the small strain and moderate rotation context are given by

$$\epsilon_{x\alpha} = u_{\alpha,x}^0 - (y - y_\alpha)v_{,xx}^0 + \frac{1}{2}(v_{,x}^0)^2. \tag{4}$$

The term involving $v^{0^2}_{,x}$ is responsible for the nonlinear geometric effects. Introducing the membrane and bending parts the strain may also be expressed as

$$\epsilon_{x\alpha} = \epsilon_\alpha^0 + (y - y_\alpha)\kappa \tag{5}$$

with $\kappa = -v_{,xx}^0$ representing the curvature.

To develop a finite element formulation for the static analysis of composite beam–columns under the above assumptions the Virtual Work Principle will be employed. For an isolated element of length ℓ, the internal virtual work is given by the sum of the contributions of the two components plus the interface connection

$$\delta W_{\text{int}} = \int_0^\ell \left(\sum_{\alpha=1,2} \int_{A_\alpha} \delta\epsilon_{x\alpha}\sigma_{x\alpha} \mathrm{d}A_\alpha + \delta s S \right) \mathrm{d}x \tag{6}$$

and the external virtual work comes from the contribution of the surface and body loads

$$\delta W_{\text{ext}} = \int_V \left(\sum_{\alpha=1,2} \delta u_\alpha\, p_{\alpha x} + \delta v\, p_y \right) \mathrm{d}V + \int_\Omega \left(\sum_{\alpha=1,2} \delta u_\alpha\, t_{\alpha x} + \delta v\, t_y \right) \mathrm{d}\Omega \tag{7}$$

with p_x, p_y denoting forces per unit volume and t_x, t_y forces per unit area.

Equality of internal and external virtual works for arbitrary compatible displacement and strain fields is equivalent to the weak form of equilibrium equations. Strong forms (i.e. differential equations) may then be devised employing integration by parts of the virtual work equation, see [2] for the linear case. From the definition of the axial strain (4), the incremental strain is given by differentiation as

$$\delta\epsilon_{x\alpha} = \delta u_{\alpha,x}^0 - (y - y_\alpha)\delta v_{,xx}^0 + v_x^0\, \delta v_x^0. \tag{8}$$

Introducing (8) in the internal virtual work expression, and defining the normal force and bending moment on each component as

$$N_\alpha = \int_{A_\alpha} \sigma_{x\alpha} \mathrm{d}A_\alpha \tag{9}$$

$$M_\alpha = \int_{A_\alpha} \sigma_{x\alpha}(y - y_\alpha) \mathrm{d}A_\alpha \tag{10}$$

the internal virtual work may be rewritten in matrix form as

$$\delta W_{\text{int}} = \int_0^\ell \delta\boldsymbol{\epsilon}^t \boldsymbol{\sigma} \mathrm{d}x \tag{11}$$

with the generalised strains and stresses given by

$$\boldsymbol{\epsilon}^T = \begin{bmatrix} \epsilon_1^0 & \epsilon_2^0 & \kappa & s \end{bmatrix} \quad \text{and} \quad \boldsymbol{\sigma}^T = \begin{bmatrix} N_1 & N_2 & M & S \end{bmatrix}. \tag{12}$$

Collecting the displacements in vector $\mathbf{u}$

$$\mathbf{u}^T = \begin{bmatrix} u_1^0 & u_2^0 & v^0 \end{bmatrix} \tag{13}$$

the generalised strains are given by

$$\boldsymbol{\epsilon} = \boldsymbol{\partial}\mathbf{u} \tag{14}$$

with matrix $\boldsymbol{\partial}$ given by

$$\boldsymbol{\partial} = \begin{bmatrix} \partial_x & 0 & \frac{1}{2}v_{,x}^0\,\partial_x \\ 0 & \partial_x & \frac{1}{2}v_{,x}^0\,\partial_x \\ 0 & 0 & -\partial_{xx} \\ -1 & 1 & h\,\partial_x \end{bmatrix}. \tag{15}$$

The generalised strain variations are given in a similar fashion by

$$\delta\boldsymbol{\epsilon} = \bar{\boldsymbol{\partial}}\delta\mathbf{u} \tag{16}$$

with matrix $\bar{\boldsymbol{\partial}}$ given by

$$\bar{\boldsymbol{\partial}} = \begin{bmatrix} \partial_x & 0 & v_{,x}^0\,\partial_x \\ 0 & \partial_x & v_{,x}^0\,\partial_x \\ 0 & 0 & -\partial_{xx} \\ -1 & 1 & h\,\partial_x \end{bmatrix}. \tag{17}$$