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Simplified method for the analysis of externally prestressed steel–concrete composite beams

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ABSTRACT

The ultimate capacity of beams prestressed with external slipping tendons cannot be evaluated by a local analysis of the critical sections and a nonlinear analysis of the whole beam-tendon structural system is required. Since this type of analysis is complex, simplified approaches are of great assistance to engineers in the design process. In this work a new simplified method is introduced for evaluating the tendon traction increment at collapse and consequently the beam flexural strength without requiring a nonlinear analysis of the whole beam-tendon structural system. A detailed description of the proposed approach is given and some applications of practical interest to externally prestressed steel–concrete composite beams are discussed.

1. Introduction

Externally prestressed steel–concrete composite members have been used since the late 1950s in buildings and bridge construction [1,2]. In addition, external post-tensioning has been extensively applied in existing bridges to reinforce damaged structures or to increase the ultimate capacity when larger than designed loads are applied [3,4]. External prestressing in steel–concrete composite beams presents many advantages [5], such as: enlarged elastic range of behaviour, increased ultimate capacity, improved fatigue behaviour, limitation of deflections, control of concrete slab cracking in the hogging regions of continuous beams, reduction of structural weight with benefits on construction economy and aesthetic value. The analysis of externally prestressed beams involves some specific issues [6–8]. Differently from conventional bonded prestressing, the stress of external tendons slipping at saddle points cannot be estimated by a local sectional analysis. A nonlinear analysis of the whole structure is required to correctly evaluate the tendon stress at failure and consequently the beam flexural strength. Since this type of analysis is complex, simplified approaches are of important help for engineers in the design process.

In the past various simplified formulas, mostly based on experimental tests, have been proposed in order to estimate the stress increment of the tendon at collapse in externally prestressed concrete beams (an extensive review of published works on this topic can be found in [9]). Many of these simplified approaches were criticized since they do not give clear information on collapse modes and in some cases predicted results are not sufficiently conservative or accurate. Recently the authors of [9] proposed a new simplified method for the analysis of concrete beams prestressed with external tendons that uses a simplified description of deformations at collapse. The proposed method is based on the observation that the shapes of the axial strain and curvature distributions at collapse do not notably change for beams with the same structural configuration, tendon path and external loads. This method can be applied to any structural configuration, tendon layout and loading condition, provided that certain values of curvature and axial strain shape at collapse are given. Differently from previous methods for externally prestressed concrete beams, the proposed approach gives a clear description of collapse modes and comprehensive information, i.e., tendon traction increment, position of the critical section where collapse occurs, collapse load multiplier and thus flexural bearing capacity. Applications to realistic beams used for concrete bridge constructions showed a very good approximation of the results obtained by nonlinear finite element analysis. This simplified approach is consistent with the conservative prescriptions of the European code [10] that allows no increase of prestressing stress at ultimate unless a nonlinear analysis of the whole beam-tendon structural system is performed. In fact the proposed simplified method can be considered as a

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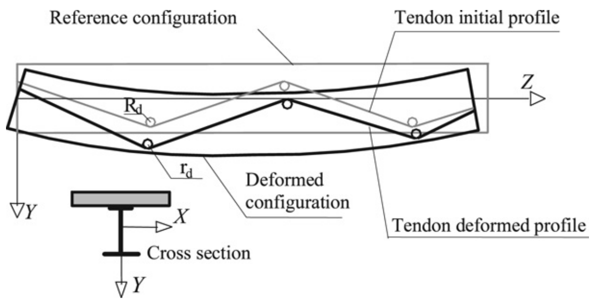

Fig. 1. Beam model with external prestressing.

simplified nonlinear analysis for the preliminary design process, leaving the more complex and accurate nonlinear analysis to the final structural verification.

While the debate over simplified methods for concrete beams with external prestressing evolved during the last decades, fewer indications were specifically provided for externally prestressed steel–concrete composite beams, mostly limited to specific cases, e.g. [2,11]. Only recently Chen and Gu [12] presented a more general approach for evaluating the incremental prestressing at the ultimate state based on a plastic analysis of the prestressed beam. Using their formulation the ultimate tendon stress in simply supported beams can be predicted as long as the ultimate deflection is determined. A different way is followed in this paper, where the simplified method presented in [9] is extended to steel–concrete composite beams. A detailed analytical description of the method is provided. Afterward comparisons between the results given by the proposed method and by a nonlinear finite element formulation [13] previously validated by comparisons with experimental tests, are illustrated. The comparisons are developed for simply supported beams with draped tendon, considering different span-to-depth ratios and span lengths.

2. Proposed method

2.1. Review of the adopted analytical model for externally prestressed beams

The proposed simplified method uses a general analytical model for externally prestressed beams with slipping tendons previously introduced [7,8] and concisely reviewed in this section for reader's convenience. The finite element formulation of this analytical model is found in [13].

A beam with symmetrical cross section is considered. An orthogonal reference frame $\{O; X, Y, Z\}$ is introduced: the Z-axis is parallel to the beam axis and the vertical plane YZ is the plane of geometric, material and load symmetry of the structure (Fig. 1). Unit vectors **i**, **j**, **k** are parallel to X, Y and Z respectively. The cross section of the beam is rigid in its own plane and remains plane and orthogonal to the beam axis after deformation. Perfect bond acts between concrete and reinforcements. External prestressing tendons are symmetrically arranged with respect to the symmetry plane YZ. Each couple of tendons is considered as one single resultant tendon with path contained in the symmetry plane. For simplicity, only one resultant tendon is considered in the following discussion. More complex tendon layouts can be considered by a straightforward extension of the model.

The previous assumptions lead to the following description of the displacement **u** of a point of the beam (i.e. only in-plane bending occurs):

$$\mathbf{u}(y,z) = v(z)\,\mathbf{j} + \left[w(z) - yv'(z)\right]\mathbf{k} \tag{1}$$

where w is the axial displacement of a reference fibre of ordinate zero and v is the vertical displacement. The only non-zero strain component in the beam is the axial strain ε

$$\varepsilon(y,z) = \varepsilon_0(z) + y\theta(z) \tag{2}$$

where $\varepsilon_0(z) = w'(z)$ is the axial strain at the reference fibre of ordinate zero in the cross-section and $\theta(z) = -v''(z)$ is the curvature. Once that general nonlinear constitutive laws are introduced for concrete and reinforcement steel, the resultants of the stress in the beam are the axial force N_R and the bending moment M_R:

$$N_R(\varepsilon_0,\theta) = \int_A \sigma(\varepsilon)\,\mathrm{d}A \tag{3}$$

$$M_R(\varepsilon_0,\theta) = \int_A y\sigma(\varepsilon)\,\mathrm{d}A \tag{4}$$

where the bending moment is computed with respect to the reference fibre of ordinate $y = 0$.

The path of the tendon is assigned by $D+1$ points where the points from 1 to $D-1$ locate the positions of the intermediate deviators and the points 0 and D locate the positions of the end anchorages:

$$\mathbf{Q}_d = y_d\mathbf{j} + z_d\mathbf{k} \tag{5}$$

where y_d and z_d $(d = 0,\ldots,D)$ are the coordinates of the d-th deviator of the tendon. It is assumed that the tendon traces a rectilinear line between two subsequent saddles. The total length of the tendon in the undeformed state is given by

$$L_t = \sum_{d=1}^{D}\left|\mathbf{Q}_d - \mathbf{Q}_{d-1}\right| = \sum_{d=1}^{D}\sqrt{[\Delta_d(y)]^2 + [\Delta_d(z)]^2} \tag{6}$$

where the operator $\Delta_d(s)$ applied to the generic scalar quantity s function of z is defined as:

$$\Delta_d(s) = s(z_d) - s(z_{d-1}). \tag{7}$$

After the deformation of the beam the deviators assume the new positions

$$\mathbf{q}_d = [y_d + v(z_d)]\,\mathbf{j} + \left\{z_d + w(z_d) - y_d v'(z_d)\right\}\mathbf{k} \tag{8}$$

where w_d and v_d are the axial and vertical components of the displacement of the d-th deviator. Consequently, the total length of the tendon in the deformed state is given by:

$$l_t = \sum_{d=1}^{D}\left|\mathbf{q}_d - \mathbf{q}_{d-1}\right| = \sum_{d=1}^{D}\sqrt{[\Delta_d(y) + \Delta_d(v)]^2 + [\Delta_d(z) + \Delta_d(w) - \Delta_d(yv')]^2}. \tag{9}$$

Assuming that the tendon can slip with negligible friction at saddle points, as happens in many real cases, e.g. [14], its strain can be calculated by the ratio between the global stretching of the tendon path and its initial length, so that it depends on the displacement field of the whole beam. Since the displacements considered in the beam description are assumed to be very small, the following linear expression is adopted for the deformed tendon length, in order to obtain a consistent formulation and to avoid making the problem nonlinear by introducing negligible terms [7,8]:

$$\varepsilon_t = \frac{l_t - L_t}{L_t} \cong \frac{1}{L_t}\sum_{d=1}^{D}\left\{\alpha_d\left[\Delta_d(w) - \Delta_d\left(yv'\right)\right] + \beta_d\Delta_d(v)\right\} \tag{10}$$