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Lateral buckling of web-tapered I-beams: A new theory

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Abstract

This paper presents a new theory for the lateral buckling of web-tapered I-beams. Linear analysis is first conducted by taking account into the tapering effects of web-tapered I-beams, where the deformation compatibilities of the two flanges and web are considered in terms of the basic assumptions of thin-walled members. Subsequently, the total potential for the lateral buckling analysis of web-tapered I-beams is developed, based on the classical variational principle for buckling analysis. The lateral buckling loads of web-tapered cantilevers and simply supported beams of I-sections from the proposed theory are compared with those from the finite element (FE) analyses using two shell element models and two widely used beam element models. The two beam element models respectively represent the equivalent method using prismatic beam elements and the typical tapered beam theory in existing literature. These comparisons show that the results based on the total potential proposed in this paper are more accurate in predicting the lateral buckling loads of web-tapered I-beams than those in existing theories, indicating that the theory proposed in this paper is superior to existing theories. It is also found that the equivalent method using prismatic beam elements may yield unreliable buckling loads of tapered beams.

Keywords: Lateral buckling; Thin-walled beams; Tapered beams; Finite element method; Shell element modelling

1. Introduction

Tapered beams are widely used in modern constructions, mainly due to their structural efficiency. At present, the web-tapered thin-walled I-beam is one of the most popular tapered beams used in practice. The strength of laterally unrestrained thin-walled beams is frequently governed by the lateral buckling (or flexural-torsional buckling) failure, and hence extensive studies were focused on the lateral buckling of thin-walled beams. Most of these studies have been concerned with prismatic beams (e.g. [11,12,14,15]), and only a few investigations are on tapered beams (e.g. [1,2,4–10,18]).

A convenient method to assess the lateral buckling load of a tapered beam is to divide such a beam into several segments and take each of these segments as an equivalent prismatic beam, as adopted by Brown [5] in the finite difference analysis. In this method, the effects of tapering could not be completely taken into account in the expressions of nonlinear strains, which may lead to incorrect lateral buckling loads [18]. Ronagh et al. [9,10] found the errors in lateral buckling loads caused by this method cannot be eliminated merely using fine mesh configuration in the finite element analysis. Note that similar treatments as in [5] are also adopted in the commercial FE software ANSYS on its tapered beam elements [3], implying that the lateral buckling load of tapered beams using tapered elements of ANSYS is not reliable, as illustrated in the later part of this paper.

Considering the effects of tapering in deriving strains, Kitipornchai and Trahair [6] built the equilibrium equations for the lateral buckling analysis of tapered beams, and numerical results were given by means of the finite difference analysis. This method was extended to tapered beams of mono-symmetric sections [7]. While Wekezer [17], Yang and Yau [18], Bradford and Cuk [4], Ronagh et al. [9], and recently Andrade and Camotim [1] investigated the lateral buckling of tapered beams employing the FE method, based on their total potentials presented. It is worth mentioning that in all literature mentioned above strains of each part of tapered beam, i.e. two flanges and web, are obtained using the same relationship between the displacements and strains, although it may be not explicitly stated. For example, the linear longitudinal strain of a point in beam, regardless of its location, is derived by differentiating the linear longitudinal displacement of this point with respect to the longitudinal axis of this beam. This

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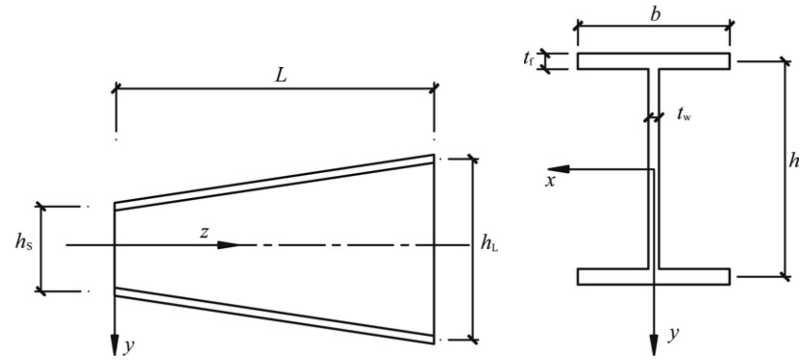

(a) Coordinate. (b) Section.

Fig. 1. Web-tapered I-beam.

methodology has been found to violate the basic assumptions of thin-walled members and may result in incorrect results in stress analysis, especially for beams of serious tapering [13]. In this paper, an attempt is made to study the lateral buckling of doubly symmetric web-tapered I-beams. The basic assumptions for thin-walled members, proposed by Vlasov [15], are adopted to derive the strain-displacement relationships for each plate of the tapered beam in order to ensure that the effects of tapering are completely considered in the framework of thin-walled theory. Based on these relationships, new equivalent sectional properties of web-tapered beams are presented. Subsequently, the total potential of web-tapered I-beams, based on the classical variational principle, is presented. Using this total potential, a finite element program is then developed for assessment of lateral buckling load of tapered beams, with which the results of the present theory are compared with those from shell element modelling and typical existing theories.

2. Deformation analysis

For a web-tapered thin-walled beam of I-section shown in Fig. 1, a right-handed coordinate system x, y and z is chosen, in which z axis coincides with the centroidal axis and x and y axes coincide with the principal axes of the cross sections. Along the longitudinal direction (z axis), the width and thickness of flanges remain constant, while the height of web varies linearly. The height of section at a distance of z from the small end is then given by

$$h = h_S + (h_L - h_S)\frac{z}{L} \quad (1)$$

in which L is length of the beam, h_S and h_L are respectively the distances between the centroids of two flanges at the small and large ends (Fig. 1(b)).

Basic deformations, including the axial deformation, bending and torsion, of web-tapered beams of doubly symmetric I-sections are studied in this section prior to the further investigations on web-tapered I-beams. In analysis, the following assumptions are adopted:

(1) Material is linearly elastic and homogeneous;
(2) Only thin-walled I-beams are concerned;
(3) Cross-sections are rigid in their own planes;

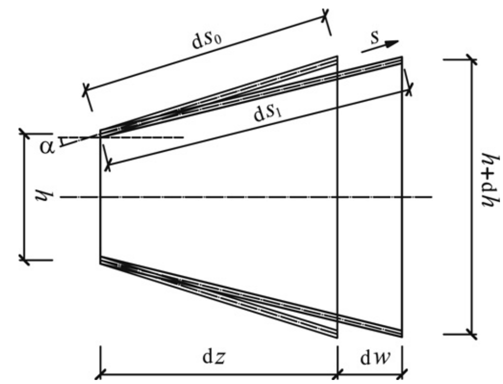

Fig. 2. Axial deformation of a tapered beam segment.

(4) The linear shear strain on the middle surface of each plate composing thin-walled beams is negligible;
(5) Deformation analyses are in the framework of small deformation theory.

2.1. Axial deformation

For a web-tapered I-beam, the centroidal axes of the two flanges (e.g. s axis of the top flange shown in Fig. 2) are not parallel to the centroidal axis of the beam, z axis, so that the deformations of the two flanges and the web of the beam are studied separately.

Fig. 2 shows the axial deformation of a web-tapered beam segment. The original length of the segment along the z axis is $\mathrm{d}z$, while the original length of the flange is $\mathrm{d}s_0$. Using the geometric relationships shown in Fig. 2, $\mathrm{d}s_0$ can be given by:

$$\mathrm{d}s_0 = \sqrt{(\mathrm{d}z)^2 + \left(\frac{\mathrm{d}h}{2}\right)^2}. \quad (2a)$$

According to the rigid profile assumption, each cross-section remains a plane in the deformed configurations, so that if this segment has an axial deformation $\mathrm{d}w$ (Fig. 2), the length of top flange becomes

$$\mathrm{d}s_1 = \sqrt{(\mathrm{d}z + \mathrm{d}w)^2 + \left(\frac{\mathrm{d}h}{2}\right)^2}. \quad (2b)$$