Available online at www.sciencedirect.com
SCIENCE DIRECT
Journal of Constructional Steel Research 62 (2006) 493–500
JOURNAL OF CONSTRUCTIONAL STEEL RESEARCH
www.elsevier.com/locate/jcsr

Effective width of a concrete slab in steel–concrete composite beams prestressed with external tendons

Shiming Chen*, Zhibin Zhang

Institute of Structural Engineering and Disaster Reduction, Tongji University, Shanghai 200092, China

Received 14 February 2005; accepted 22 August 2005

Abstract

The finite element method is applied to evaluate the effective width of a concrete slab in composite beams prestressed with external tendons. The beams studied are simply supported, with external prestressed strands straightly and draped arranged. The effective width for the prestressed composite beams evaluated based on the finite element parametric study is compared with that of plain composite beams without prestressing, and code provisions. The influence of the shear connections on the effective width and the shear slips along the beam span, as well as the incremental stresses developed in the prestressed strands, are analyzed. The behaviors of the external prestressed composite beams subject to the shrinkage and creep in the concrete slab are also discussed.

Keywords: Effective width; Composite beam; Prestressed; External tendons; Shear connection

1. Introduction

Steel–concrete composite beams prestressed with high strength external tendons have demonstrated many advantages as compared with normal non-prestressed composite beams and have been used in building and bridge construction for years. Literature can be reviewed in the design of the Bonners Ferry Bridge [1] and the California existing highway bridge upgrading [2]. A typical form of composite construction consists of a slab connected to a series of parallel steel members. The structural system is therefore essentially a series of interconnected T-beams with wide, thin concrete flanges. In such a system, the concrete flange width may not be fully effective in resisting compression due to 'shear lag'. The concept of effective width is generally adopted as a simplification of the shear lag problem. The effective width b_{eff} is defined as 'that width of slab which acted on by the actual maximum stress would cause the same static effect as the variable stress which exists in fact'.

Research shows that the ratio b_{eff}/B depends not only the relative dimensions of the system, but also on the type of loading, the support conditions and the cross section considered. For the stress control in the serviceability state, the deformability of the shear connection between the concrete slab and the steel beam plays an important role, but is generally neglected in the early evaluation of the effective width. A recent experimental study by Amadio et al. [3] demonstrated that the effective width increases with load, approaching the width of the whole slab near the collapse. However, in most codes of practice, very simple formulae are given for the calculation of effective width, though this may lead to some loss of economy. For simply-supported beams in buildings, the European Recommendations [4] proposed in the Commentary that the effective width is $L/8$ on each side of the steel web, but not greater than half the distance to the next adjacent web, nor greater than the projection of the cantilever slab for edge beams. The width of steel flange occupied by shear connectors, a can be also added. These limits are also given in the bridge code BS5400: Part 5 [5], but in the Chinese code provision [6] for design of steel–concrete composite beams, the effective width of $L/6$ on each side of the steel web is suggested.

For composite beams prestressed with external tendons, however, restricted to the composite beam configuration, in practice, the external prestressed strands are normally arranged close to each side of the steel beam; this raises the question of what effective width of the concrete flange the external prestress force should account for. Obviously, a wider breadth of concrete

* Corresponding author.
E-mail address: chensm@mail.tongji.edu.cn (S. Chen).

0143-974X/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jcsr.2005.08.009

Nomenclature

a_0	connection width
a_{ei}	half effective width minus half connection width
A_s, A_c, A_i, A_{ps}	area of cross sections for steel beam, concrete slab and composite beam and steel strands
b_{eff}, b_{ei}	effective width and half effective width
B	available width of concrete slab
e	eccentricity of the tendons to the centroid of the cross section
E_s, E_c, E_{ps}	elastic modulus of steel, concrete and steel strands
f_y	steel yield strength
I_s, I_c, I_i	second moment of area for concrete, steel and composite sections
n, n_f	available number of shear connectors and required number of shear connectors capable of a full shear connection
M, M_c, M_s	moments exerted on composite section, concrete and steel portions
N, N_c, N_s	axial force on the composite section, concrete slab and steel section
N_0	initial prestress force
P_d	shear strength capacity of an individual connector
ΔT	incremental tendon force
V_u	ultimate longitudinal shear force
α_e	coefficient of effective width
η	modular ratio of materials
γ_n	shear connection degree
ν	Poisson coefficient
ψ	coefficient of creep of concrete
σ, σ_{max}	normal stress, the maximum normal stress of concrete
$\Delta\sigma_{ps}$	incremental stresses in the external tendons

flange leads to a lower prestress, and narrower breadth leads to a higher prestress, and that will all affect the final design output.

An important feature of these prestressed composite beams is that under variable loading, incremental stress will develop in the tendons, which has a secondary bending action on the beam, and in turn affects slippage at the interface of the steel and concrete as well as the effective width of the concrete flange.

In this paper, the effective concrete width of composite beams prestressed with external tendons is investigated based on a finite element analysis. A parametric study was carried out to evaluate the conventional expression of effective width justified for the prestressed composite beams. The influence of the shear connections on the effective width and the shear slips along the beam span, as well as the stress increment of prestressed cable, are analyzed. The behaviors of the external prestressed composite beams subject to shrinkage and creep in the concrete slab are also discussed.

2. Effective width of concrete for prestressed composite beams

The concept of effective width is introduced for a composite beam in the hypothesis of linear-elastic component materials. It is the slab width that, for a uniform distribution of normal stresses equal to the maximum stress in the concrete slab, gives the same resultant as the one due to the real stress distress distribution, which can be expressed as the following:

$$b_{eff} = \frac{\int_0^B \sigma \, dx}{\sigma_{max}} \tag{1}$$

where b_{eff} is effective width of concrete slab, B is real width of slab, σ is the real normal stress distribution across the slab and σ_{max} is the maximum normal stress in the slab.

Consider the bending moment and axial forces due, for example, to prestressing acting on the composite beam, to be the resultant of *sectional forces* acting on the individual steel and concrete components. These sectional forces will consist of couples M_s and M_c and axial forces N_s and N_c acting on the steel and concrete components as shown in Fig. 2. For equilibrium, the following equations can be derived:

$$N_c + N_s = N \tag{2}$$

$$M_c + M_s + N_s(y_{ci} + y_{si}) = M. \tag{3}$$

Assume the cross section remain plane and there is no vertical separation between the steel beam and the concrete slab; the bending moments and axial forces shared by the steel and concrete components can be derived as the follows:

$$N_c = N\frac{\eta A_c}{A_i} + M\frac{\eta A_c}{I_i} y_{ci} \tag{4}$$

$$M_c = M\frac{\eta I_c}{I_i} \tag{5}$$

$$N_s = N\frac{A_s}{A_i} + M\frac{A_s}{I_i} y_{si} \tag{6}$$

$$M_s = M\frac{I_s}{I_i} \tag{7}$$

where N and M are resultant of these *sectional forces*; I_c, I_s and I_i are the second moment of area for concrete, steel and composite sections respectively; A_c, A_s and A_i are the areas for concrete, steel and composite sections; η expressed as $\eta = E_s/E_c$ is a modular ratio of materials, where E_s is elastic modular of steel and E_c is elastic modular of concrete; y_{ci} is the distance between the centroids of the concrete section and the composite section; and y_{si} is the distance between the centroids of the steel section and the composite section as shown in Fig. 2.

From Eq. (4), there are two portions contributing to the internal force in the concrete slab, which can be expressed as the following:

$$N_c = \int_{A_c} \sigma_N \, dA + \int_{A_c} \sigma_M \, dA \tag{8}$$

where σ_N and σ_M are normal distributed stress contributed from the axial force and bending moment exerted on the composite section.