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Non-uniform torsion of stiffened open thin-walled members of steel structures

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Abstract

A general solution is derived, from distributional differential equations of equilibrium, for thin-walled structural members that are non-uniformly torsioned continuous, or situated in a Winkler rotational-elastic medium, with constant open cross-sections, stiffened with transversal ribs along their length. The equations were solved for a general case by means of the Laplace transform. Solutions were obtained in the form of closed generalized functions, holding good — for both non-uniform torsion displacements and internal forces and their influence lines — along the entire length of the bars. The effect of ribbing (diaphragms and closed ribs) on constrained torsion displacements and internal forces and on the respective influence lines is assessed for single-span and continuous I-section structural members.

The results of non-uniform torsion tests carried out on I-bars stiffened along their length with battens or closed ribs (Fig. 12), in order to verify the theoretical models of the stiffened members (particularly with battens), are discussed. The test results confirmed the poor effectiveness of stiffeners in the form of battens directly joining the flanges of I-section members (Fig. 12(a)), due to the deformation of the flanges. The deformation can easily be eliminated by modifying the battens so that they join the flanges to the web (Fig. 12(b)). Tests showed that, by using the modified battens, one can effectively stiffen open thin-walled members. The effectiveness of the stiffening that is obtained is similar to that of stiffeners in the form of closed ribs.

Keywords: Steel structure; Thin-walled continuous member; Open cross-section; Non-uniform (warping) torsion; Continuous and point bracing; Transversal rib; Batten plate; Distribution calculus; Model testing

1. Introduction

General analytical solutions for straight open thin-walled members, proposed by, among others, Vlasov in [1], arrived at through the integration of the problem differential equations, cover only unbraced single-span members and the simplest cases of loading. Attempts at solving more complex cases are rather futile, because of the integration ranges that follow from the load, the bracings and the indirect support conditions. Therefore methods similar to those used in the theory of bent bars are applied in order to solve more complex problems. A three-bimoment method, a five-bimoment method and a force method are proposed in, respectively [1–5], an initial parameter method in [1] and [4], a displacement method in [2] and [3], and Cross's method in [6] and [7]. Also, work [8] should be mentioned here in which selected torsion problems are solved by exploiting the analogy between a torsioned thin-walled bar and a bent and tensioned bar.

Advances in computer technology have greatly contributed to the development of numerical methods for solving open thin-walled bar torsion problems. The methods can be applied to both unbraced bars (e.g. [9] and [10]) and bars with local bracings along their length (e.g. [11] and [12]), for which solutions worked out by classical methods are known (e.g. [13,14] and [15]). It is also proposed to apply numerical methods (e.g. [10,16]) to the more complex 2nd-order problems considered in many earlier papers.

The above numerical methods of solving non-uniformly torsioned, open thin-walled members with constant cross-sections when applied to more complex systems are more effective than the integration of the problem differential equation presented in monograph [1]. The solutions, however, are exclusively numerical, which may be inconvenient when a static quantity is to be analysed. In such a case, general analytical solutions are the most convenient.

Below, an effective method of working out a general analytical solution is presented for simple prismatic, open, thin-walled bars by solving a bar torsional angle differential

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equation in the distributional form (e.g. [17]). The method can be used to determine torsional angle functions, and thus static quantities and influence lines, for an arbitrarily loaded single- or multi-span thin-walled bar with point bracings. The solution has the form of a generalized function which holds good for the entire length of the bar. The method is an expansion and generalization of the concept put forward in Nowiński's paper [18]. It was previously presented in [19] and [20] and, recently, in a broader context in book [21]. Distribution calculus was also applied to selected open thin-walled member spatial stability problems (e.g. [22] and [23]).

2. Distributional solution to thin-walled member torsion problems

2.1. Structural members in Winkler rotational-elastic medium

The well-known differential equation for torsional angles $\varphi(z)$ of the cross-section relative to shear centres axis S for a thin-walled structural member with any open cross-section, situated in a Winkler rotational-elastic medium, has the form [1]:

$$EI_\omega\varphi^{(4)} - GI_t\varphi^{(2)} + K_\varphi\varphi = n(z), \tag{1}$$

where E and G are, respectively, Young's modulus and a modulus of elasticity in shear, I_ω and I_t are, respectively, a sectoral moment of inertia and a pure torsional moment of inertia, K_φ is the coefficient of elasticity of the rotational-elastic medium, and $n(z)$ is the torsional load.

Despite the fact that it was derived in [1] for the classic function formulation.

Eq. (1) is also valid in the distribution domain. This follows from distribution theory (e.g. [17,21]), which introduced the notion of distribution as a generalization of the notion of function. In the distributional form of Eq. (1), $n(z)$ is the given distribution, while $\varphi(z)$ is the sought distribution.

If a proper load distribution is substituted for $n(z)$ in differential equation (1) for a single-span thin-walled bar whose scheme and loading are shown in Fig. 1, the equation assumes the form [19]:

$$\begin{aligned}EI_\omega\varphi^{(4)} - GI_t\varphi^{(2)} + K_\varphi\varphi &= B\delta^{(1)}(z - z_B)\\ &+ M_z\delta(z - z_M) + m_z\left[h(z - z_{m1}) - h(z - z_{m2})\right],\end{aligned} \tag{2}$$

where $\delta(z - z_i)$ and $h(z - z_i)$ are Dirac and Heaviside distributions, and $\xi^{(k)} = \mathrm{d}^k\xi(z)/\mathrm{d}z^k$ $(\xi = \varphi, \delta)$.

Eq. (2) is a non-homogenous differential equation of order 4, with constant coefficients. Its solution, arrived at through the Laplace transformation, is the following distribution [19]:

$$\begin{aligned}\varphi(z) = {}& \varphi_0\sum_{i=1}^{4}\frac{s_i^2 - 2k_t^2}{4(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i z} + \varphi_0^{(1)}\sum_{i=1}^{4}\frac{s_i^2 - 2k_t^2}{4s_i(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i z}\\ &+ \varphi_0^{(2)}\sum_{i=1}^{4}\frac{1}{4(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i z} + \varphi_0^{(3)}\sum_{i=1}^{4}\frac{1}{4s_i(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i z}\\ &+ \frac{B}{EI_\omega}\sum_{i=1}^{4}\frac{1}{4(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i(z - z_B)}h(z - z_B)\\ &+ \frac{M_z}{EI_\omega}\sum_{i=1}^{4}\frac{1}{4s_i(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i(z - z_M)}h(z - z_M)\\ &+ \frac{m_z}{EI_\omega}\left\{\left[\frac{1}{k_\varphi^4} + \sum_{i=1}^{4}\frac{1}{4s_i^2(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i(z - z_{m1})}\right]\right.\\ &\times h(z - z_{m1})\\ &\left.- \left[\frac{1}{k_\varphi^4} + \sum_{i=1}^{4}\frac{1}{4s_i^2(s_i^2 - k_t^2)}\,\mathrm{e}^{s_i(z - z_{m2})}\right]h(z - z_{m2})\right\},\end{aligned} \tag{3}$$

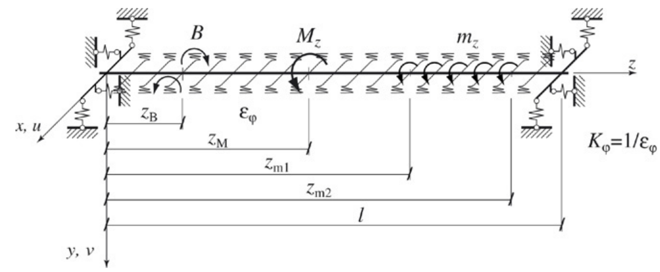

Fig. 1. Thin-walled bar in rotational-elastic Winkler medium.

where

$$2k_t^2 = \frac{GI_t}{EI_\omega}, \quad k_\varphi^4 = \frac{K_\varphi}{EI_\omega} = \frac{1}{\varepsilon_\varphi EI_\omega}, \tag{4}$$

and points

$$s_{1,2,3,4} = \pm\sqrt{k_t^2 \pm \sqrt{k_t^4 - k_\varphi^4}}$$

are the poles of the transform of Eq. (2).

Depending on the relations between coefficients k_t and k_φ, solutions for special cases can be worked out from the general solution (3).

Integration constants φ_0, $\varphi_0^{(1)}$, $\varphi_0^{(2)}$, $\varphi_0^{(3)}$ in solution (3) are determined from the bar support conditions and the generalized internal forces from the well-known differential relations [1–5].

In the case of the considered thin-walled members, point bracings, which limit the torsion of the selected intermediate cross-sections of the bars, are much more common in practice than the Winkler rotational-elastic medium. Moreover, the transversal ribs joined to the flanges and the web, used in open thin-walled members, by constraining the warping of the member's cross-section affect the non-uniform torsion displacements and internal forces. This fact should not be overlooked in calculations, particularly in the case of closed ribs.

The following section presents a general solution to the problem of the non-uniform torsion of continuous bars with a constant open cross-section and transversal ribs spaced pointwise along the length, arrived at using the distributional method.

2.2. Ribbed multispan thin-walled members

The differential equation for the torsional angles of the cross-section relative to the axis of shear centres for a multispan