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Recent advances on postbuckling analyses of thin-walled structures: Beams, frames and cylindrical shells

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Abstract

The lateral postbuckling response of thin-walled structures such as bars and frames with members having steel rolled shapes as well as circular cylindrical shells under axial compression is thoroughly reconsidered. More specifically via a simple and very efficient technique it is found that beams with rolled shapes (symmetric or non symmetric) under uniform bending and axial compression exhibit a stable lateral–torsional secondary path with limited margins of postbuckling strength. New findings for the static and dynamic stability of frames with crooked steel members – due to the presence of residual stresses – are also reported. It is comprehensively established that the coupling effect due to initial crookedness and loading eccentricity may have a beneficial effect on the load-carrying capacity of the frames. Moreover, simple mechanical models are proposed for simulating the buckling mechanism of axially compressed circular cylindrical shells. Very recently Bodner and Rubin proposed an 1-DOF mechanical model whose buckling parameters correlated to those of the shells by using an empirical formula based on experimentally observed shell buckling loads. In the present analysis a new 2-DOF model for the static and dynamic buckling of axially compressed circular cylindrical shells, which can include mode coupling, is presented.

Keywords: Thin-walled structures; Beams; Frames; Cylindrical shells; Postbuckling analysis; Static/dynamic load; Imperfections; Residual stresses; Rolled shapes; Warping

1. Introduction

In the last decades considerable attention has been given to the effect of nonlinearities on the stability of structural systems. This is justified by the fact that application of a linear (bifurcational) analysis may fail to predict the actual (critical) buckling load. Indeed, this is true when the critical (bifurcation) point is unstable implying a serious reduction of the load-carrying capacity in case of the existence of initial geometric imperfections. Such imperfections always exist in steel structures due to the presence of residual stresses in steel beams with rolled shapes. On the other hand, one should note that the majority of actual systems, if accurately modeled, exhibit a limit point instability rather than bifurcational buckling. This requires a complete nonlinear stability analysis for estimating the exact buckling load and the associated load-carrying capacity.

The present analysis dealing with thin-walled structures focuses attention on steel beams and frames with members consisting of rolled shape cross-sections as well as on circular cylindrical shells under axial compression. Reviewing the state-of-the-art up to 1994, to the best knowledge of the author, there was not any nonlinear stability (postbuckling) analysis of steel beams or beam–columns with rolled shape cross sections (either non symmetric or symmetric) subjected to lateral or lateral–torsional buckling. Indeed, the existing studies referred to linear analyses, which cannot give information about the stability of the critical bifurcational state.

During the last decade Kounadis and his associates presented several studies dealing with the postbuckling analyses of simply supported steel beams or beam–columns with rolled shapes subjected to lateral or flexural–torsional buckling including the effect of the cross-section warping. Such analyses based on a nonlinear expression for the curvature led to the establishment of the initial postbuckling equilibrium path. The following were discussed: (a) cases with doubly symmetrical cross-sections [1] and with monosymmetric cross-sections [2–4], which fail either through flexural (Euler) buckling or

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flexural–torsional (non-Eulerian) buckling, and (b) cases with open asymmetric thin-walled cross-sections [5], which always fail by flexural–torsional buckling. Using a simple and very efficient (approximate) analytic technique [6] it was established that the critical bifurcational state is stable with a very shallow postbuckling path with very limited margins of postbuckling strength. Thus there is no imperfection sensitivity and the critical buckling load defines the actual load-carrying capacity.

Subsequently, the **static and dynamic** buckling of a two-bar steel frame with initially crooked bars (due to residual stresses), which is subjected to an eccentrically applied load at each joint was properly studied. The individual effect of imperfection sensitivity for static and dynamic loading of the initial (crookedness) curvature as well as of the loading eccentricity was fully assessed. Moreover, by a thorough discussion of the combined effect of these two imperfection parameters, the unexpected result was found that a suitable arrangement of the bar initial curvature in the frame configuration, may lead to its maximum load-carrying capacity associated with an asymmetric bifurcation [7,8]. Such a beneficial (stabilizing) effect of imperfections on the buckling load is also experienced in case of a dynamic (step) loading [9,10].

The third type of thin-walled structures refer to the postbuckling analysis of circular cylindrical shells under axial compression. Early studies on these shells have shown that their actual load-carrying capacity can be safely obtained only via a nonlinear analysis since in some cases classical (linear) analyses lead to completely unacceptable results. On the other hand, extensive tests have shown that the theoretically predicted minimum postbuckling (peak) load is unrealistically **low** to be useful as a guide for designers for all but the very thin shells. The *discrepancy* between *test and theory* rendered necessary application of Koiter's or other theory combined with empirical results. According to Koiter's theory, the limit point (peak) load for a circular cylindrical shell depends on Poisson's ratio and mainly on small geometric imperfections, whose variety and measurement has been the subject of numerous papers [11,12].

Recently, Calladine [13] proposed an *empirical formula* for the limit point (peak) load based on test results. A possible way of reconciling Koiter's load with Calladine's empirical formula is to assume that geometric imperfections in Koiter's analysis depend on the ratio (t/R) of thickness t to the radius R of the shell.

Despite the existence of more than 2000 studies devoted to the buckling of axially compressed thin-walled circular cylindrical shells, the relative research continues in an attempt to improve existing treatments. A principal reason for this interest is that the geometric imperfections have not as yet been fully identified. According to test observations by Singer et al. [14,15] it was deduced that the onset of buckling in the above shells is highly localized over a small region. Thereafter, the buckling deformation assumes a diamond-shaped pattern, which is progressively repeated in neighbouring sections, while the load drops suddenly at a fixed overall shortening. It was also found that the membrane response due to such localized buckling is nonlinear and analogous to a nonlinear (soft) spring. Very recently, Bodner and Rubin [16] on the basis of testing emptied beer cans concluded that the buckling process is that of a local longitudinal strip supported at its middle by an elastic foundation which exhibits snap-through caused by shallow arch-like behavior in the circumferential direction. Bodner and Rubin presented an interesting re-examination of this problem by using a single mode mechanical model [similar to that of [17–19]] which shows some of the salient buckling characteristics of uniformly axially compressed circular cylindrical shells.

In the present paper, being an extension of the last work by Bodner and Rubin, two mechanical models are proposed for modeling the buckling of axially compressed circular cylindrical shells under static and dynamic (step) loading. The first one, much simpler than the single model of Bodner and Rubin, *neglects* the change of the bar length without any practical effect on the accuracy of the obtained results [20]. The second mechanical model of 2-DOF (two degrees of freedom) can include the mode coupling effect [9,21]. Numerical examples for the three types of the above thin-walled structures illustrate the simplicity, efficiency and reliability of the methodology proposed herein.

2. Mathematical analysis

In this section attention will be focused on the description of the basic equations governing the postbuckling response of the above three types of elastic thin-walled structures. Since this analysis is addressed to engineers not necessarily familiar with rudiments of the mathematical aspects of stability, some care has been taken to avoid pertinent technicalities. However, more details on the mathematical procedure can be found in the related references cited in the text.

2.1. Bars with rolled shape cross-sections

Consider the simply supported bar with an open thin-walled asymmetric cross-section shown in Fig. 1 which is subjected to a compressive centrally applied load P. For such a general type of cross-section (whose centroid C does not coincide with the shear center S) the failure always occurs by *flexural–torsional* instability. Subsequently, the formulation and notation used by Timoshenko and Gere [22] is adopted. Denoting by x and y the principal centroidal axes of the cross-section, x_o and y_o are the coordinates of the shear center S. Consider the bar in a slightly deformed configuration associated with a *translation* and a *rotation* of the cross-section (Fig. 1). The translation is defined by the deflections u (along the axis x) and υ (along the axis y) of the shear center S as well as of the centroid C. Namely, the shear center moves from S to S', while the centroid moves from C to C'. The rotation of the cross-section about the new position of the shear center S' is denoted by φ and the final position of the centroid by C'' [3,5]. Since, the total deflections of the centroid C are $u+y_o\varphi$ and $\upsilon-x_o\varphi$ the bending moments of the simply supported bar due to the central thrust P are $M_y = P(u + y_o\varphi)$ and $M_x = P(\upsilon_o - x_o\varphi)$. One can show also that the twisting moment M_{zz} is equal to [23]

$$M_{zz} = P y_o \frac{\mathrm{d}u}{\mathrm{d}z} - P x_o \frac{\mathrm{d}\upsilon}{\mathrm{d}z} + \frac{P}{A} I_p \frac{\mathrm{d}\varphi}{\mathrm{d}z} \tag{1}$$

where A is the area of cross-section and $I_p = I_x + I_y$.