Available online at www.sciencedirect.com
SCIENCE DIRECT®
Journal of Constructional Steel Research 62 (2006) 592–604
JOURNAL OF CONSTRUCTIONAL STEEL RESEARCH
www.elsevier.com/locate/jcsr

Approximate formulae for natural periods of plane steel frames

C. Chrysanthakopoulos, N. Bazeos, D.E. Beskos*

Department of Civil Engineering, University of Patras, GR-26500 Patras, Greece

Received 24 November 2004; accepted 28 September 2005

Abstract

Approximate formulae for determining by hand with a high enough accuracy the first three natural periods of vibration of plane steel unbraced and braced frames are provided. These formulae are based on the modeling of a plane steel frame as an equivalent cantilever beam for which analytical expressions for the natural periods are available. Extensive parametric studies involving the finite element computation of the first three natural periods of 110 plane steel unbraced and braced frames are employed to establish correction factors for the equivalent beam modeling formulae which are functions of the number of stories and bays of the frame. The resulting corrected formulae permit a highly accurate determination of the first three natural periods of plane steel frames.

Keywords: Natural periods; Plane steel frames; Braced frames; Unbraced frames; Multi-storey frames; Multi-bay frames; Equivalent flexural-shear beam

1. Introduction

Seismic building codes, such as UBC [1] or EC 8 [2], provide analytical expressions for the computation of the design seismic acceleration in terms of the natural period of vibration of the structure. Thus, the design base shear can be computed easily by either multiplying this acceleration by the structural mass if only the first mode participates in the response, or by modal synthesis if the first few modes participate in the response. In the first case, a knowledge of the first (fundamental) period is necessary, while in the second case a knowledge of the first few natural periods of the structure is required. Lopez and Cruz [3] have established empirical formulae providing the required number of the first few modes necessary for obtaining the seismic response of building frames with a relative error of 5% and 10%, while seismic building codes [1,2] require the participation in the response of so many modes as to have at least a participation of 90% of the total structural mass.

Seismic codes, such as UBC [1] or EC 8 [2], provide very simple but crude empirical formulae for the fundamental period of structures in terms of their material (steel or reinforced concrete), structural type (frame, shear wall, etc.) and height. Goel and Chopra [4], on the basis of experimental data gathered from eight earthquakes, were able to improve the accuracy of these formulae by modifying their coefficients.

A more rational way of constructing formulae for the hand computation of natural periods of vibrations of tall plane wall-frame buildings is to establish relations to model those buildings as equivalent flexural-shear cantilever beams for which the natural periods are available in analytical form in standard structural dynamics texts. One can mention here, e.g., the works of Coull [5], Rosman [6], Rutenberg [7], Stafford Smith and Crowe [8], Li et al. [9] and Zalka [10]. The advantage of those formulae is that they enable the designer to rapidly compute natural periods by hand during the preliminary design stage. This advantage is not present in those methods of determining natural periods of large order plane or space frames and trusses by modeling them as continuous beams and employing the finite element method. In those methods the goal is to drastically reduce the computational work and not to provide simple formulae for the periods. For a comprehensive review on the subject, one can consult Noor [11].

In this work approximate formulae for determining by hand the first three natural periods of vibration of plane steel frames are presented. Formulae of high enough accuracy for both unbraced and braced frames are based on them being modeled as equivalent cantilever beams, in accordance with the approach of Stafford Smith and Crowe [8] for which analytic expressions for natural periods are available. These formulae are modified

* Corresponding author. Tel.: +30 2610 996559; fax: +30 2610 996579.
E-mail address: d.e.beskos@upatras.gr (D.E. Beskos).

0143-974X/$ - see front matter
doi:10.1016/j.jcsr.2005.09.005

by some correction factors, functions of the number of frame stories and bays, which are constructed with the aid of extensive parametric studies involving finite element computation of the first three natural periods of 110 plane steel unbraced and braced frames. Thus, the present work is analogous to that of Goel and Chopra [4] in the sense that, through data from numerical experiments, improvements in the accuracy of existing formulae are realized. In this work, frames of up to 15 stories are considered and thus the first three modes are enough to satisfy the 90% vibrating mass criterion of seismic codes [1, 2] as well as the empirical formulae of Lopez and Cruz [3].

2. Free vibrations of a flexural-shear beam

The free vibrations of a flexural-shear prismatic beam are governed by the equation [8]

$$EI\frac{\partial^4 v}{\partial x^4} - EI(\alpha k)^2\frac{\partial^2 v}{\partial x^2} + m\frac{\partial^2 v}{\partial t^2} = 0 \tag{1}$$

where EI and m are the flexural rigidity and mass per unit length, respectively, of the beam, $v = v(x,t)$ is the lateral deflection of the beam, x and t denote axial coordinate and time, respectively, and αk expresses a shear rigidity to be defined explicitly later on.

The above equation can be decomposed into the equivalent system of the two equations

$$EI\frac{\partial^4 v_f}{\partial x^4} + m\frac{\partial^2 v_f}{\partial t^2} = 0 \tag{2}$$

$$EI\frac{\partial^4 v_{sf}}{\partial x^4} - EI(\alpha k)^2\frac{\partial^2 v_{sf}}{\partial x^2} + m\frac{\partial^2 v_{sf}}{\partial t^2} = 0 \tag{3}$$

for which

$$\begin{aligned} &v = v_1 + v_2 \\ &v_1 = [(k^2-1)/k^2]v_f, \qquad v_2 = (1/k^2)v_{sf} \end{aligned} \tag{4}$$

with v_f denoting the deflection component due purely to flexural motion and the v_{sf} deflection component due to a coupled shear-flexural motion.

For a cantilever beam of length H with the fixed end at $x = 0$ and the free end at $x = H$, the boundary conditions read [12]

$$\begin{aligned} &v(0,t) = 0, \qquad \partial v(0,t)/\partial x = 0 \\ &V(H,t) = 0, \qquad M(H,t) = 0 \end{aligned} \tag{5}$$

where V and M denote shear force and bending moment, respectively. Thus, Eq. (2), describing flexural free vibrations, yields the eigenvalue equation [12]

$$1 + \cos\lambda_f H \cosh\lambda_f H = 0 \tag{6}$$

whose solution in terms of its eigenvalues λ_f is [12]

$$\begin{aligned} &(\lambda_f H)_1 = 1.875, \qquad (\lambda_f H)_2 = 4.694 \\ &(\lambda_f H)_n \cong (n-0.5)\pi, \qquad n = 3, 4, \ldots \end{aligned} \tag{7}$$

where

$$\lambda_f^4 = \omega_f^2 m/EI \tag{8}$$

with ω_f being the natural frequency of flexural vibration. Eq. (3) on the other hand, describing coupled flexural-shear free vibrations, yields the eigenvalue equation [7]

$$\begin{aligned} &2 + \left[\left(\frac{\lambda_1}{\lambda_2}\right)^2 + \left(\frac{\lambda_2}{\lambda_1}\right)^2\right]\cos\lambda_1 H \cosh\lambda_1 H \\ &\quad + \left[\frac{\lambda_2}{\lambda_1} - \frac{\lambda_1}{\lambda_2}\right]\sin\lambda_1 H \sinh\lambda_2 H = 0 \end{aligned} \tag{9}$$

for which

$$\lambda_2^2 = \lambda_1^2 + (\alpha k)^2, \qquad \lambda_1^2\lambda_2^2 = \lambda_{sf}^4 = \omega_{sf}^2 m/EI \tag{10}$$

with ω_{sf} being the natural frequency of coupled flexural-shear vibrations.

According to Rutenberg [7] and the Southwell–Dunkerley approximation, one can finally obtain the natural periods T of free vibrations governed by Eq. (1) in the form

$$T = (2\pi/\lambda^2)\sqrt{m/EI} \tag{11}$$

where

$$\frac{1}{\lambda^4} \approx \left(\frac{k^2-1}{k^2}\right)\frac{1}{\lambda_f^4} + \left(\frac{1}{k^2}\right)\frac{1}{\lambda_{sf}^4}. \tag{12}$$

Approximate, yet reasonably accurate, solutions of Eq. (9) have the form [8]

$$(\lambda_{sf}H)^2 \cong (\lambda_f H)^2[1 + (k\alpha H/\lambda_f H)]^{1/2}, \qquad k\alpha H < 6 \tag{13}$$

$$(\lambda_{sf}H)^2 \cong (n-0.5)\pi(1 + k\alpha H), \qquad k\alpha H \geq 6. \tag{14}$$

3. Plane frame structures as equivalent beams

Plane orthogonal, braced or unbraced frames, shear walls or coupled frame-wall systems fixed on the ground can be modeled as equivalent flexural-shear cantilever beams. This equivalence can be established by expressing EI and αk of Eq. (1) in terms of the geometrical and material parameters of the frame or the frame-wall system. Following [8] one has that, for

$$\alpha = [GA/EI]^{1/2}, \qquad k = [1 + (EI/EAc^2)]^{1/2} \tag{15}$$

the equivalent flexural-shear beam can be established provided the three parameters EI, EAc^2 and GA can be expressed in terms of the properties of the frame or the frame-wall system.

Thus, for an unbraced bay of a frame, EI and EAc^2 of the equivalent beam can be expressed as [8]

$$EI = \sum_{j=1}^{n}(EI)_j, \qquad EAc^2 = \sum_{j=1}^{n}(EAc^2)_j \tag{16}$$

where $(EI)_j$ is the flexural rigidity of the jth vertical member (column or wall) of the system, $(EA)_j$ the axial rigidity of the jth vertical member, $(c)_j$ the distance of the jth column to the center of the area of the vertical members of the lateral load resisting frame, and n the total number of vertical members of the frame. On the other hand, GA of the equivalent beam can