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Energy-efficient train control: The two-train separation problem on level track☆

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ABSTRACT

When two trains travel in the same direction along the same track it is a common safety requirement that they must be separated by at least one signal. If the signals are located at fixed positions, they divide the track into separate sections and the safety requirement means that two trains cannot occupy the same section at the same time. Safe separation can be ensured by specifying supplementary section clearance times which define the latest allowed exit time for the leading train and the earliest possible entry time for the following train. The clearance times could initially be based on an existing timetable but we will show that adjusting these times can substantially decrease the total energy required by the trains.

In this paper we find driving strategies that minimize total energy consumption and allow both trains to finish on time while adhering to the separation constraints imposed by the supplementary clearance times. We establish a new necessary condition to check whether a set of specified clearance times is optimal and discuss a heuristic procedure to find the optimal clearance times and the corresponding speed profiles. We illustrate our methods with a simplified but realistic case study.

1. Introduction

When two trains travel in the same direction along the same track it is a common safety requirement that they must be separated by at least one signal. The signals, located at fixed positions along the track, divide the track into different sections. Safe separation constraints mean that two trains cannot occupy the same section of track at the same time. We wish to find driving strategies for each train that minimize the total energy consumption and allow the trains to complete their journeys at the specified time while maintaining adequate separation.

Our aim is to understand the driving strategies that enable safe separation and to find a general form for the optimal speed profiles of the leading train and the following train. In practice, the problem of train separation is obscured by a myriad of additional constraints. We deliberately exclude complications which may undermine the structural integrity of the

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underlying theory. Thus we restrict our attention to identical trains on journeys with no intermediate stops. We do not allow overtaking and we do not consider timing constraints imposed at designated locations to meet trains on other lines or to pass trains travelling in the opposite direction.

In general a following train must not enter a section until the leading train has cleared that section. Furthermore, the following train must enter the section at a speed that allows it to stop within the section, if necessary. In practice this means that a train travelling at full speed may need to be more than one section behind the leading train. This paper describes a method that can be used with sections of any length and is easily applied to situations where more than one vacant section is required for safe separation. Our method can also be modified to simulate continuous block signalling by choosing very short fixed sections of equal length and insisting that the trains be separated by a fixed number of vacant sections at all times. We elaborate on these adaptations later in the paper.

The classic *single-train control problem* is to minimize the energy required to drive a train from one station to the next within a given time. It is well-known (e.g. Howlett (2000); Khmelnitsky (2000); Liu and Golovitcher (2003); A.R. Albrecht et al. (2013a)) that when the train starts and finishes at rest, on level track the optimal strategy is made up from an ordered sequence using phases of *maximum power*, *holdspeed using power* (also referred to as *speedhold*), *coast* and *maximum brake*. The problem of finding the optimal sequence of driving modes and the optimal switching locations has been solved for a single train on general track with continuously varying gradient and speed limits. Gradients, in particular, introduce significant complications. The key to energy-efficient driving is the concept of an optimal holdspeed. For each given journey duration and known initial and final speeds, there is a unique holdspeed for the entire journey that completely determines the optimal strategy (Howlett et al., 1994; Howlett, 1996, 2000; Khmelnitsky, 2000; Liu and Golovitcher, 2003; A. R. Albrecht et al., 2013a).

Fig. 1 shows an optimal speed profile for a single train on level track travelling from $x = 0$ to $x = 144{,}000$ m with a specified journey duration of $T = 7200$ s. The train starts and finishes its journey with speed $v(0) = v(X) = 0$. The optimal driving strategy on level track consists of four phases: *maximum power* from $x = 0$ to $x = 1059$ m, *speedhold* at speed $V = 20.41$ ms^{-1} from $x = 1059$ m to $x = 137{,}180$ m, *coast* from $x = 137{,}180$ m to $x = 143{,}760$ m and *maximum brake* from $x = 143{,}760$ m until the end of the journey.

Now consider what should happen if two trains travel in the same direction on the same track. If the trains are sufficiently far apart (e.g. if the first train finishes its journey before the second train begins its journey), then each train can be driven using the optimal strategy for a single train and the total energy used will be minimized. However, for trains travelling sufficiently close to each other the use of identical strategies may mean that safe separation is not maintained. In other words, separation constraints may mean that at least one of the trains will not be able to follow a speed profile that minimizes energy use. Indeed the overall strategy may be better if neither train follows the single-train optimal profile. Despite a comprehensive theoretical analysis in the literature of the single-train problem (see Section 2), there has been almost no corresponding work on a theory of optimal driving strategies for two or more trains travelling in the same direction on the same track where safe separation is necessary at all times.

In this paper we address the two-train separation problem on level track with specified starting and finishing times for each journey. By restricting our attention to level track, we avoid the complications caused by steep grades. One significant benefit is that we can obtain analytic expressions for the distance travelled, elapsed time and cost of each driving mode. We will assume that the track consists of a number of sections separated by fixed signals. For safe separation we will not allow two trains to occupy the same section at the same time; that is, the following train cannot enter a particular section until the leading train has left it. We can achieve this by specifying intermediate signal times (equivalently, intermediate section clearance times). Each intermediate time is used to define both an exit time for the leading train on a particular section and an entry time for the following train on the same section. For any given set of prescribed intermediate times we will find optimal speed profiles for both the leading train and the following train that respect the specified section clearance times. A more challenging mathematical problem, which we also address, is to find a set of intermediate clearance times—an optimal set—that minimizes the total energy consumed. The ultimate aim of this research is construction of energy-efficient timetables for many trains on a corridor with driving strategies specified for each train.

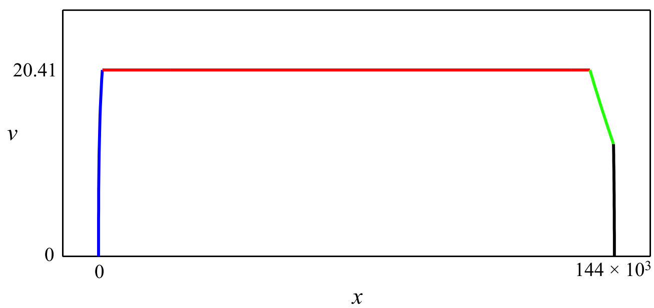

Fig. 1. Optimal speed profile $v = v(x)$ (in ms^{-1}) for a single train on level track where x denotes position (in m).