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The influence of jaw's curvature on the results of the Brazilian disc test

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ABSTRACT

The general contact problem of a disc squeezed between jaws of arbitrary curvature is considered employing Muskhelishvili's complex potentials. Taking advantage of the general solution introduced, the closed-form expressions for the stresses along strategic loci (loaded rim, loaded diameter, disc's center) are obtained, in terms of the ratio ρ of the disc's to the jaw's curvature. Then, the effect of ρ (as well as that of the relative stiffness of the disc's and jaw's materials dictating the contact arc) on the stress distribution along these loci is explored. It is concluded that, for both smooth contact (zero friction) and contact with friction, the role of the jaw's curvature is significant not only along the disc-jaw contact arc (as it could be expected), but also all along the loaded diameter. On the other hand, it is indicated that the stress field at the disc's center is more or less insensitive to the jaw's curvature assuming that ρ lies within the range (0, 0.67) or in other words within the limits defined by the two standardized suggestions, i.e. that of American Society for Testing and Materials (ASTM) (plane loading platens with $\rho = 0$) and that of International Society for Rock Mechanics (ISRM) (curved jaws with $\rho = 0.67$). The upper limit of this range is a kind of compromise between the need to make the stress field at the disc's center independent of the boundary conditions while keeping at the same time the contact angle large enough to reduce the stress concentration and the risk for premature fracture initiation far from the disc's center. For jaws with radius of curvature exceeded by that suggested by ISRM, the stress field at the disc's center is significantly influenced. Especially for jaws with radius approaching that of the disc, the stress field at the disc's center is dramatically distorted rendering Hondros' formula inapplicable and the test results erroneous.

1. Introduction

Among the most controversial topics, that seriously concern scientists and engineers using the Brazilian disc test for the indirect determination of the tensile strength of brittle materials, is the load transfer technique, i.e. the way the specimen is placed on, fixed to and loaded by the loading frame. Even today, more than seventy years after the test was proposed by Akazawa (1943) and Carneiro (1943), a definite and unique answer is not available; even the two standardized techniques, i.e. that proposed by the American Society for Testing and Materials (ASTM) (ASTM D3967-08, 2008) and that suggested by the International Society for Rock Mechanics (ISRM) (ISRM, 1978), adopt different ways for loading the disc-shaped specimens.

In general, it is accepted (either implicitly or directly) that, independently of the way the load is applied to the specimen, the stress state at the disc's center corresponds to a biaxial stress field with $k = \sigma_{\text{compressive}}/\sigma_{\text{tensile}} = 3$. In other words, it is accepted that the actual boundary conditions, prevailing along the disc's loaded rims, do not seriously influence the stress state at the disc's center. This was indicated long ago by Peltier (1954) who concluded that as long as the contact arc is not too large (less than one fifth of the disc's diameter) the tensile stresses remain uniform across the major part of the loaded diameter. Similar conclusions were drawn by Barenbaum and Brodie (1959) and Rudnick et al. (1963). The above certainty was recently confirmed (at least for the ISRM suggested standard) by Markides and Kourkoulis (2013), who derived analytic solutions for the stress field all over the disc taking into account the actual boundary conditions (distribution of radial and frictional stresses along the disc's loaded rims), as they were obtained from the solution of the respective disc-jaw elastic contact problem (Kourkoulis et al., 2012).

Unfortunately, the fact that the stress field at the disc's center is more or less insensitive to the actual conditions along the loaded rims does not constitute a necessary and sufficient condition for the validity of the results obtained from the Brazilian disc test. It was

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long ago indicated by many researchers that, unless certain precautions are taken, it is quite possible that the results of the test may be erroneous; this is the case when fracture starts away from the disc's center, i.e. in the immediate vicinity of the disc-jaw or the disc-platen interface.

Fairhurst (1964) was perhaps the first one who quantitatively discussed the validity of the test with respect to the boundary conditions. Adopting an empirical generalization of Griffith's criterion, he studied analytically the role of the loaded rim's length, assuming that the solution of Hondros (1959) is valid. He concluded that, increasing the contact angle, fracture could start away from the disc's center and he proposed an optimum semi-contact angle α equal to about arctan(1/8). Moreover, he highlighted the crucial role of the tangential stresses (friction) developed along the loaded rim. The latter was thoroughly studied by Addinall and Hackett (1964), who concluded that its influence is negligible for the central two thirds of the disc's loaded diameter. They also considered the role of soft packing pieces which are interposed between the disc and the loading platen to assist better load distribution. They pointed out that the test results were more consistent in the absence of such inserts.

The most exhaustive investigation of the effect of boundary conditions on the results of the Brazilian disc test was carried out by Mellor and Hawkes (1971). In their milestone paper, they studied also the influence of the actual distribution of radial stresses, indicating that it should not be considered uniformly. Moreover, they proposed possible ways for reducing the contact (friction) stresses. In this context, they suggested, for the first time, the use of curved jaws of radius exceeding that of the specimens, rather than of plane loading platens. The use of jaws of radius (curvature) equal to that of the specimen (proposed a few years earlier by Jaeger and Hoskins (1966)) was rejected since "… *under load, the edges of the platen grooves bit into the rock specimen and promoted platen cracking*". Along these lines of thought and based on Timoshenko's solution for the elastic contact of two cylindrical discs (Timoshenko and Goodier, 1970), they designed cylindrical jaws with radius equal to about 1.5 times the radius of the specimens. Their suggestion became the basis of the respective ISRM (1978) standard. Their conclusions were soon supported by similar ones drawn by Hudson et al. (1972), who used "*radiused end-caps*" of curvature equal to that of the tested discs "… *to distribute the load over a known arc (10°)*" and found that it was not possible "… *to obtain a Brazilian test specimen in which failure started from the center of the disc*".

In the light of the above conclusions, it becomes evident that, although the stress field at the disc's center is indeed insensitive to the actual boundary conditions, the reliability of the results of the Brazilian disc test strongly depends on the actual conditions prevailing along the disc-jaw (disc-platen) interface, i.e. along the disc-jaw contact arc that is gradually developed as the load imposed by the loading frame increases. Given that the actual conditions along the contact arc are dictated (among others) by the curvature of the disc and that of the jaw, an attempt is here described to quantify the influence of this factor on crucial aspects of the Brazilian disc test.

2. Laboratory implementation of the Brazilian disc test and some open questions

Concerning the simulation of the boundary conditions prevailing along the disc-jaw or the disc-loading platen contact arc, the questions that are even today open, are related to the influence of a number of parameters on the stress field developed in the disc (both at its center and also in the vicinity of the disc-jaw contact area). These parameters are: (i) the length of the contact arc; (ii) the actual distribution of radial stresses along the contact arc; and (iii) the role of shear (friction) stresses inevitably developing along the contact arc in case of nonperfectly smooth contacting areas.

Although the above aspects were first brought to the light of scientific research more than half a century ago (Fairhurst, 1964), definite conclusions are not yet drawn and the problem is still under intensive study (see for example the recently published paper by Japaridze (2015)).

It is evident that, in order for an answer to the above questions to be given, one should consider the way the Brazilian disc test is implemented in praxis. Generally speaking, there are two approaches:

(i) Curved jaws (Fig. 1a). In case the ratio of the disc's radius to that of the jaw is 0.67, it corresponds to the standardized procedure suggested by ISRM (1978). A special case of curved jaws is the one shown in Fig. 1b (introduced by Hudson et al. (1972) and recently used by Erarslan and Williams (2012) and by Komurlu and Kesimal (2015)), where the jaws and the specimen have the same radius.
(ii) Planar loading platens (Fig. 1c), which is in fact the procedure standardized by ASTM (ASTM D3967-08, 2008). There are some variances of this configuration, as it is shown for example in Fig. 1d (a soft insert is placed between the disc and the platen) or Fig. 1e (small metallic cylinders are interposed between the disc and the loading platen).

A special configuration widely used nowadays is the flattened Brazilian disc (Fig. 1f) proposed by Wang et al. (2004), whose scientific team contributes intensively also in the direction of improving the standardized cracked Brazilian disc test (Wang et al., 2013).

Within the framework of the present study, attention will be paid to the role of the relative curvature of the jaw with respect to that of the disc and therefore the flattened Brazilian disc will not be considered. Along the same lines, the configurations of Fig. 1d (flat loading platens with soft inserts) will be excluded from the present study since the role of friction on the specific "three-material" complex (metallic platen-soft insert-brittle disc) is not as yet clarified in a satisfactory manner, contrary to the "two-material" complex (metallic jaw-brittle disc), for which the role of friction is more or less well understood (Addinall and Hackett, 1964; Hooper, 1971; Lavrov and Vervoort, 2002; Lanaro et al., 2009; Markides et al., 2012; Kourkoulis et al., 2013). Concerning the configuration of Fig. 1e, it corresponds in fact to cylinder-to-cylinder contact and it is represented by the respective one of Fig. 1a.

In the context of the above discussion, the configuration to be studied consists of a circular disc of finite radius R_D which is compressed between two jaws of arbitrary radius of curvature R_J (Fig. 2a) ranging from $R_J = R_D$ (disc and jaw have the same radius, Fig. 1b) to $R_J \rightarrow \infty$ (flat loading platens, Fig. 1c). Two cases will be considered: one corresponding to perfectly smooth contact and one corresponding to contact with finite non-zero coefficient of friction between the jaw and the disc. The topic will be here confronted as a contact problem within the framework of linear elasticity. The purpose of the study is to analytically explore the influence of the relative curvature, as it is expressed by the ratio $\rho = R_D/R_J$, on the stress field developed, with focus on the two critical regions: the disc's center and the contact arc. As soon as the general problem is solved, two specific cases will be considered in juxtaposition: one corresponding to $\rho = 0.67$ (which represents the ISRM suggested procedure) and $\rho \rightarrow 0$ (which represents the ASTM procedure). Some additional characteristic ρ-values will be also considered for the sake of completeness.

The main advantage of the present study is that it considers the "disc-jaw" complex rather than an isolated disc loaded by an