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Settlement predictions of footings on sands using probabilistic analysis

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ABSTRACT

The design of footings on sands is often controlled by settlement rather than bearing capacity. Therefore, settlement predictions are essential in the design of shallow foundations. However, predicted settlements of footings are highly dependent on the chosen elastic modulus and the used method. This paper presents the use of probabilistic analysis to evaluate the variability of predicted settlements of footings on sands, focusing on the load curve (predicted settlements) characterization. Three methodologies, the first- and second-order second-moment (FOSM and SOSM), and Monte Carlo simulation (MCS), for calculating the mean and variance of the estimated settlements through Schmertmann (1970)'s equation, are presented and discussed. The soil beneath the footing is treated as an uncorrelated layered material, so the total settlement and variance are found by adding up the increments of the layers. The deformability modulus (E_{Si}) is considered as the only independent random variable. As an example of application, a hypothetical case of a typical subsoil in the state of Espirito Santo, southeast of Brazil, is evaluated. The results indicate that there is a significant similarity between the SOSM and MCS methods, while the FOSM method underestimates the results due to the non-consideration of the high-order terms in Taylor's series. The contribution of the knowledge of the uncertainties in settlement prediction can provide a safer design.

1. Introduction

Probabilistic or reliability analysis provides a mean of evaluating the combined effects of uncertainties and a way of distinguishing conditions with high or low uncertainties (Duncan, 1999). In geotechnical design, it has become increasingly popular in the last decade (Sivakugan and Johnson, 2004), since the geotechnical analysis based on conventional deterministic approaches, using safety factors, is highly dependent on the models and input parameters.

However, most common studies in probabilistic analysis published in the literature discuss the ultimate limit state (ULS), representing the failure probability of a foundation (bearing capacity criterion), even considering that the settlement criterion is often more critical in the design of shallow foundations, especially for foundation with the width greater than 1 m (Schmertmann, 1970; Rezania and Javadi, 2007) or 1.5 m (Das and Sivakugan, 2007).

Several publications have shown that the predicted settlements of footings on sands are highly dependent on the methods used (Tan and Duncan, 1991; Sivakugan and Johnson, 2004). Fig. 1 shows a comparison of settlement predictions, made by 11 methods based on standard penetration test (SPT) results, with measured settlements. Tan and Duncan (1991) concluded that, through the high variability obtained, there is a tradeoff between accuracy and reliability.

Moreover, the settlement predictions are also influenced by the subsoil spatial variability due to a combination of different geological, environmental and physico-chemical processes (Phoon and Kulhawy, 1999).

This paper presents the use of probabilistic analysis to evaluate the settlements of footings on sands, focusing on the load effect curve (predicted settlements) characterization. Three methodologies, the first- and second-order second-moment (FOSM and SOSM), and Monte Carlo simulation (MCS), for calculating the mean and variance of the estimated settlements through Schmertmann (1970)'s equation, are presented and discussed. As an example of application, a hypothetical case in the state of Espirito Santo, southeast of Brazil, is evaluated.

2. Probabilistic analysis

It is intuitive to believe that the predicted settlement values (total and differential) of foundation are influenced by the

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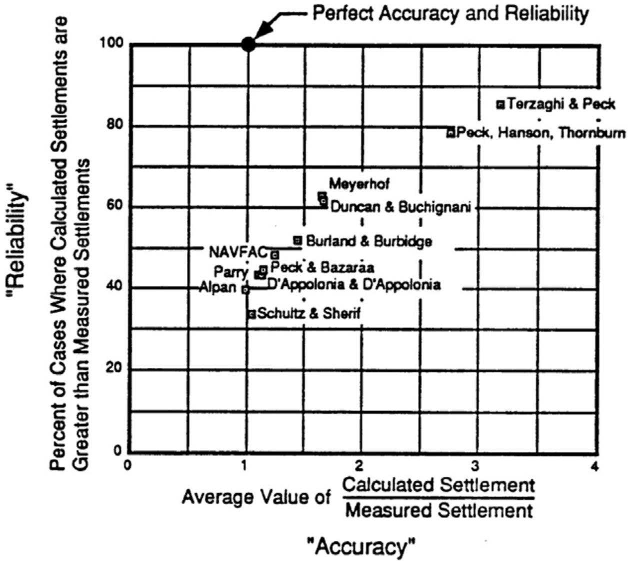

Fig. 1. Relation between accuracy and reliability of settlement predictions made by 11 methods based on SPT results (Tan and Duncan, 1991).

variability of adopted soil parameters, which affect the reliability of further design decisions (Fenton et al., 1996). Fig. 2 demonstrates the examples of two different settlement prediction cases. The predicted mean values of the settlement are 15 mm and 20 mm for cases A and B, respectively. In a traditional deterministic analysis, the prediction made for case A would be considered safer (lower comparative value). However, when considering the variability of the predictions, represented by the dispersion of the probability density curves, it is clearly observed that the probability of the predicted settlement for the case that exceeds a preset limit value of 25 mm (the shaded area below the curve) is larger for case A than that for case B, which indicates that the case B is more reliable.

Generally, the failure probability of a foundation, p_E, at its serviceability limit state (SLS) is a function of the relative position and scatter degree of the density curves of the load effect $\rho(x)$, representing the variability of the predicted settlements, and the resistance $\rho_{lim}(x)$, representing the variability of the limiting settlement, as shown in Fig. 3:

$$p_E = \int_0^{\infty} \rho(x)\rho_{lim}(x)dx \tag{1}$$

The settlement predictions of footings on sands are usually made by traditional methods (e.g. Schmertmann, 1970; Schmertmann et al., 1978; Burland and Burbidge, 1985; Berardi and Lancellotta, 1991). Limiting settlements evaluation can be made by using observational, empirical, structural or numerical modeling methods (Negulescu and Foerster, 2010).

The present paper focuses on the load effect curve (predicted settlement) characterization, assuming that the variability of the resistance curve (limiting settlement) is null for simplification. In other words, it is considered constant for some specific deterministic values, as the examples discussed in Fig. 1. Thus, the probability of occurrence of limiting settlements, p_E, becomes

$$p_E(\rho \geq \rho_{lim}) = \int_{\rho_{lim}}^{\infty} \rho(x)dx \tag{2}$$

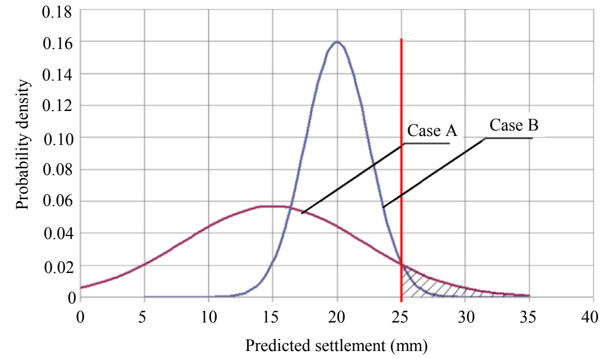

Fig. 2. Example of comparative analysis for two cases (A and B) of predicted settlement with different variability degrees.

The integrals of Eqs. (1) and (2) are commonly solved by using analytical approximations (or reliability methods). Here, three methodologies, i.e. FOSM, SOSM, and MCS, for calculating the mean and variance of the predicted settlements through Schmertmann (1970)'s equation are briefly presented and discussed as a simple and practical way to characterize the settlement solicitation curve for a case of a single footing on sandy soil.

3. Methodologies

Schmertmann (1970)'s equation is briefly presented in Eq. (3) as it is one of the most popular methods for settlement predictions, commonly discussed in geotechnical engineering text books. It is a simple semi-empirical equation, based on the theory of elasticity and supported by model tests and finite element analysis, to predict the settlement of a footing on granular soil. The soil is proposed to be divided into sublayers, which are considered to be elastic, homogeneous and isotropic, with constant deformability modulus, E_{Si}. A simplified strain influence factor, I_z, was introduced and its distribution with depth was defined. Such a factor is basically dependent on the ratio of the depth to the foundation width (z/B) and can be evaluated by graphical or equational forms. The maximum soil strain occurs at a depth $z = B/2$ under the footing embedment depth, D_f, and decreases linearly until the depth equals $2B$, where the strain can be ignored. No distinction is made between square or strip footings.

$$\rho = C_1 C_2 \sigma^* \sum_{i=1}^{N} \left(\frac{I_{zi} \Delta_{zi}}{E_{Si}} \right) \tag{3}$$

where $\sigma^* = \sigma - q$ is the net footing pressure, σ is the applied footing pressure, q is the pressure due to soil mass at the depth D_f,

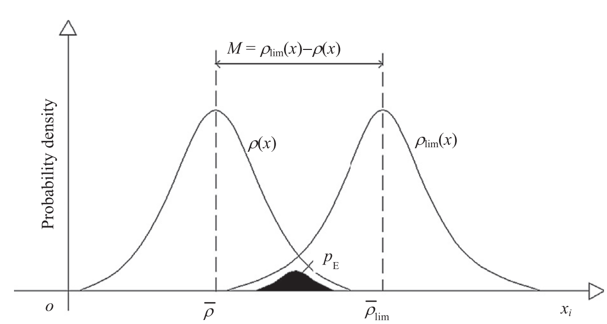

Fig. 3. Reliability analysis of a foundation at the SLS: Solicitation (predicted settlement) and resistance (limit settlement) probability density curves.