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Effects of porosity on seismic velocities, elastic moduli and Poisson's ratios of solid materials and rocks

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ABSTRACT

The generalized mixture rule (GMR) is used to provide a unified framework for describing Young's (E), shear (G) and bulk (K) moduli, Lame parameter (λ), and P- and S-wave velocities (V_p and V_s) as a function of porosity in various isotropic materials such as metals, ceramics and rocks. The characteristic J values of the GMR for E, G, K and λ of each material are systematically different and display consistent correlations with the Poisson's ratio of the nonporous material (ν_0). For the materials dominated by corner-shaped pores, the fixed point at which the effective Poisson's ratio (ν) remains constant is at $\nu_0 = 0.2$, and $J(G) > J(E) > J(K) > J(\lambda)$ and $J(G) < J(E) < J(K) < J(\lambda)$ for materials with $\nu_0 > 0.2$ and $\nu_0 < 0.2$, respectively. $J(V_s) > J(V_p)$ and $J(V_s) < J(V_p)$ for the materials with $\nu_0 > 0.2$ and $\nu_0 < 0.2$, respectively. The effective ν increases, decreases and remains unchanged with increasing porosity for the materials with $\nu_0 < 0.2$, $\nu_0 > 0.2$ and $\nu_0 = 0.2$, respectively. For natural rocks containing thin-disk-shaped pores parallel to mineral cleavages, grain boundaries and foliation, however, the ν fixed point decreases nonlinearly with decreasing pore aspect ratio (α: width/length). With increasing depth or pressure, cracks with smaller α values are progressively closed, making the ν fixed point rise and finally reach to the point at $\nu_0 = 0.2$.

1. Introduction

Porosity is a fundamental microstructural parameter for most natural and man-made materials and influences significantly physical properties of these materials such as diffusion coefficient, elastic wave velocities, elastic moduli, Poisson's ratio, yield, rupture or ductile strength, thermal conductivity, electrical conductivity, fluid permeability, dielectric constant, magnetic permeability. Sediments and rocks (e.g. soils, sands and sandstone) are typical examples of natural porous materials. The main goal of this study is to calibrate the porosity-dependence of seismic velocities (e.g. P- and S-wave velocities: V_p and V_s), V_p/V_s ratio or Poisson's ratio, and elastic moduli of isotropic solid rocks because such a calibration is required to interpret correctly the geophysical data of natural resources (e.g. petroleum and natural gases). Such a calibration is also helpful to the understanding of the mechanical properties of geotectonical engineering materials that are equally porous. Furthermore, foamed metals, sintered ceramics, hollow concretes and cellular polymers are man-made porous materials that have been widely used for thermal and acoustic insulation, impact energy absorption, vibration suppression, air or water filtration, fluid flow control, self-lubricating bearing, floatation and lightweight components. Therefore, to model accurately the mechanical properties of candidate solid materials in terms of their component properties, porosity and microstructure have broad significance for a wide range of fields from materials engineering to Earth sciences.

In the present study, our particular attention is paid to the influence of porosity on the compressional (P) and shear (S) wave velocities (V_p and V_s) as well as the Poisson's ratio, which is a function only to the ratio of V_p/V_s, of isotropic solid materials and rocks. The reason for this purpose is simple and given below. Direct observations on the nature of the materials that constitute the Earth are limited to studies of surface outcrops and rocks that have been obtained from mining and drilling. Drilling for mining and scientific purposes has penetrated to generally a few kilometers and the maximum to 10–12 km (10 km for the KTB hole in Germany, and 12 km for the Kola hole in Russia) beneath the surface, leaving much of the Earth's interior inaccessible. Much of our knowledge of the chemical composition, physical state and structure of the Earth's interior mainly comes from seismic data. Interpretation of these seismic data, in turn, is largely constrained by the extrapolation of laboratory-measured seismic properties of

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relevant rocks thought to exist in a given geological and physical (i.e. pressure and temperature) environment. Apart from chemical composition, phase transformation, metamorphic reaction, dehydration, partial melting, temperature and pressure (e.g. Ji et al., 2002), porosity is a critical factor to affect the elastic wave properties of dry and wet rocks. However, the studies of this kind are extremely difficult because the geometrical shape, size distribution and connectivity of pores in three-dimensional, opaque rocks are generally unknown. Thus, the results from laboratory studies on the dependence of the elastic and seismic properties of isotropic man-made materials (e.g. metals, ceramics, and oxides) with known porosities may help to better understand seismic data from the natural rocks.

The elastic properties of an isotropic material or rock can be described by any two of the four elastic moduli termed Young's modulus (E), shear modulus (G), bulk modulus (K) and Lame parameter (λ). The Young's modulus (E) is defined as the ratio of the stress along an axis over the strain along that axis in the range of uniaxial stress in which Hooke's law holds. The bulk modulus (K) measures a substance's resistance to uniform compression while the shear modulus (G) is defined as the resistance to a simple shear strain that produces a shape change without changing total volume. The Lame parameter (λ) is quite special because it relates stresses and strains in perpendicular directions (Jaeger, 1969). The physical meaning of λ can be clearly illustrated in a special case of uniaxial strain where $\varepsilon_1 \neq 0$, and $\varepsilon_2 = \varepsilon_3 = 0$ (i.e. no displacement occurs in the direction perpendicular to the x-axis): $\lambda = \sigma_2/\varepsilon_1 = \sigma_3/\varepsilon_1$ (Ji et al., 2010). Goodway (2001) believed that λ is closely related to material's incompressibility ($\lambda = K - 2G/3$) and contains a higher proportion of information about the resistance to a change in volume caused by a change in pressure. The above 4 parameters (i.e. E, G, K and λ) are the most intrinsic elastic coefficients to express stress in terms of strain.

The most common geophysical parameters measurable are compressional (P) and shear (S) wave velocities (V_p and V_s) and densities (ρ) of elastic media. E, G, K and λ for isotropic elasticity can be easily determined from the measured seismic data:

$$V_\mathrm{p} = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \tag{1}$$

$$V_\mathrm{s} = \sqrt{\frac{G}{\rho}} \tag{2}$$

$$\nu = \frac{E}{2G} - 1 \tag{3}$$

$$K = \frac{E}{3(1 - 2\nu)} \tag{4}$$

$$\lambda = K - \frac{2}{3}G \tag{5}$$

where ν is the Poisson's ratio. $\lambda \leq 0$ if $V_\mathrm{p}/V_\mathrm{s} \leq \sqrt{2}$. $\lambda/G \geq 1$ if $V_\mathrm{p}/V_\mathrm{s} \geq \sqrt{3}$ (e.g. almandine, antigorite, calcite, dolomite, fayalite, feldspar, hematite, hornblende, lizardite, sillimanite, spinel, spessartine, talc, rutile, and wustite), and $0 \leq \lambda/G < 1$ if $\sqrt{2} \leq V_\mathrm{p}/V_\mathrm{s} < \sqrt{3}$ (e.g. quartz, bronzite, diallage, enstatite, forsterite, sapphire, periclase, and staurolite). The information about G and λ can be extracted from the inversion of P- and S-wave reflectivities (Estabrook and Kind, 1996; Goodway et al., 1997; Gray and Andersen, 2000; Goodway, 2001; Dufour et al., 2002; Gray, 2003; Li et al., 2003). Obviously, any two of these four moduli (E, G, K and λ) offer the most fundamental parameterization of elastic seismic waves to extract information about the composition and structure of rocks in the Earth's interior. In the literature, however, only E, K, G and ν are usually reported although λ is also an intrinsic and invariant property of elastic media under given conditions. So far, little systematic research work has been carried out on the characterization of λ values for crystalline rocks and materials (Ji et al., 2010). Here we also calibrate the porosity-dependence of λ values for the solid materials of interest.

Poisson's ratio (ν) is the negative of the ratio of transverse strain to the axial strain when an isotropic material is subjected to uniaxial stress. For an isotropic material at a given temperature and a given pressure, ν is a constant which lies between -1 and 0.5. Materials with $\nu < 0$ are called auxetic materials because there is an increase in volume when compressed (Lakes, 1987). Here only the Poisson's ratios of isotropic materials are considered, which can be calculated from P- and S-wave velocities (V_p and V_s) in the isotropic material:

$$\nu = 0.5 - \frac{0.5}{(V_\mathrm{p}/V_\mathrm{s})^2 - 1} \tag{6}$$

Among the elastic properties, Poisson's ratio is the least studied, but at the same time the most interesting only (Christensen, 1996; Gercek, 2007; Ji et al., 2009; Wang and Ji, 2009). For example, Poisson's ratio is a helpful hint to overcome the non-uniqueness of the interpretation of either V_p or V_s alone in terms of petrological composition (e.g. Ji et al., 2013a,b; Shao et al., 2014). The crustal Poisson's ratio information can be obtained from the analysis of teleseismic receiver functions using single station techniques (Clarke and Silver, 1993; Zandt and Ammon, 1995; Ji et al., 2009). The interpretation of such crustal Poisson's ratio results (e.g. Owens and Zandt, 1997; Chevrot and van der Hilst, 2000; Nair et al., 2006) has been largely based on an assumption that Poisson's ratio depends primarily on SiO_2 content (with more mafic rocks corresponding to higher ν values) and fluid content (Tarkov and Vavakin, 1982; Christensen, 1996; Owens and Zandt, 1997). As shown in Fig. 1a, the common silicate rocks form an arc-shaped trend line indicating that the Poisson's ratio increases with density as the lithology changes from granite, schist, felsic gneiss, through diorite-syenite, intermediate gneiss and metasediment, to gabbro-diabase, amphibolite, and mafic gneiss, and then decreases as the rocks become ultramafic in composition (i.e. pyroxenite and peridotite). However, the monomineralic rocks such as quartzite, serpentinite, anorthosite, limestone and marble are significantly deviated from the trend line (Ji et al., 2009). Sandstone has a chemical composition similar to quartzite but significantly a higher ν value than quartzite, indicating that ν increases with increasing porosity since $\nu = 0.08$ for quartz (Fig. 1b). The effects of porosity on the Poisson's ratio for the other types of natural rocks have not been studied in detail due to the lacking of experimental data over a wide range of porosities and pore geometry. Thus, one of the main goals of this investigation is to constrain the effects of porosity on the $V_\mathrm{p}/V_\mathrm{s}$ ratio or the Poisson's ratio of porous materials and rocks.

2. Generalized mixture rule (GMR)

In the present study, we use the generalized mixture rule (GMR) (Ji, 2004; Ji et al., 2004) to model the variations of seismic velocities and elastic moduli as a function of porosity. The GMR is expressed as

$$M_\mathrm{c}^J = \sum_{i=1}^{N} \left(V_i M_i^J \right) \tag{7}$$