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Calibrating partial factors for Danish railway embankments using probabilistic analyses

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ABSTRACT

High costs are connected with upgrading railway embankments throughout Denmark using the partial factors for geotechnical design calibrated for general application. One way to reduce the costs is reliability-based calibration of the partial factors to a reasonable safety level taking into account the specific design situations and uncertainties relevant to railway embankments. A reliability-based design has been investigated, resulting in an optimal partial factor for the considered subsoil. With a stochastic soil model to simulate the undrained shear strength of soft soil deposits, the partial factor is calibrated using asymptotic sampling for the reliability assessment. The calibration shows that the partial factor can be reduced significantly compared to the value specified in the Danish National Annex to DS/EN 1997-1 (2007), Eurocode 7.

1. Introduction

In Denmark, many existing railways are upgraded to higher train velocities and more tracks, leading to increasing demands concerning the structural safety. With the current railway codes, a railway embankment which is upgraded with an extra track must fullfil consequence class CC3 in Eurocode 7 (Banedanmark, 2010). These requirements are expensive, making embankments adequately safe but not excessively safe.

The partial factor in the Danish National Annex to DS/EN 1997-1 (2007) has been calibrated for application to a wide range of structures. In Denmark, the partial factor for the undrained shear strength is rather high, $\gamma_{c_u} = 1.8K_{FI}$, compared to the general factor proposed in Eurocode 7, $\gamma_{c_u} = 1.4K_{FI}$, where the factor K_{FI} accounts for the severity of the potential failure. This leads the embankment design to become very expensive.

Using the partial factor method (DS/EN 1990, 2007) for specific design of railway embankments, special considerations can be taken into railway design. The partial factor method ensures a certain safety level for the structure, but this level is unknown to the designer since he/she has no influence on the selected partial factors proposed in the Danish National Annex to Eurocode 7. Specifying the partial factor makes it easy for the designer to introduce safety in the structure, but if the partial factors are not chosen with care, the structure could end up being too safe, and thus very expensive.

Rather than defining a set of partial factors to be used in the design process, a required level of safety could be defined. This will make the design process more complex, but also more flexible allowing the designer to choose a set of partial factors that will ensure the required safety level for the structure. This could be accomplished using probabilistic design of the embankment (Ching et al., 2011). Here all parameters are modelled by stochastic variables with a distribution function, a mean value and a standard deviation (Koudelka, 2011).

The concept of probabilistic design is presented in the paper. Probabilistic modelling of soil and loads are described, and a sampling technique is provided to minimize calculation time. In the analyses conducted by Lodahl et al. (2012), it was found that the current general demands in the Danish National Annex to Eurocode 7 are too conservative for a railway embankment in cohesive fill situated on soft soils.

The analyses presented are based on the utilization of circular slip surface, and two embankments designed with the optimized partial factor are investigated. Furthermore, modelling with the purpose of estimating the most probable slip surfaces for each of the embankments is carried out.

The analyses are based on a typical situation when upgrading existing railway tracks in Denmark, namely embankments built

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with cohesive clay fill situated on soft soils, are often peat or gyttja. The embankments are often old, with consolidation effects resulting in harder soils under the embankment than that beside it. In the presented analyses, embankments of different heights, situated on different thicknesses of soft soils, are analysed.

2. Probabilistic modelling

In the probabilistic design method, a *limit state equation* is formulated. This equation is in general formulated as the load bearing capacity or resistance, R, minus the loading, L. Each of these can be modelled as dependent on a number of stochastic variables:

$$g(\boldsymbol{x}) = R(\boldsymbol{x}_R) - L(\boldsymbol{x}_L) \tag{1}$$

where $\boldsymbol{x}_R$ and $\boldsymbol{x}_L$ contain the uncertain parameters applied in model R and L, respectively; and $\boldsymbol{x} = (\boldsymbol{x}_R,\boldsymbol{x}_L)$. When the outcomes of the limit state equation are positive, i.e. $g(\boldsymbol{x}) > 0$, the structure is 'safe', whereas a negative limit state equation means failure. The probability of failure, P_f, is calculated as the probability of the limit state equation being negative, and is linked to the *reliability index*, β, by

$$\beta = \Phi^{-1}\left(-P_f\right), \quad P_f = \int_{g(x)>0} f_X(x)\mathrm{d}x \tag{2}$$

where $P_f = P(g(\boldsymbol{x}) \le 0)$. The theoretical background for structural reliability can be found in various literature, e.g. Madsen et al. (1986) and Ditlevsen and Madsen (1996).

The integral in Eq. (2) may be solved by first order reliability method (FORM) or simulation, of which the latter will be discussed in this paper. The use of failure simulation techniques is generally recommended when the limit state equation is discontinuous. Since the determination of the resistance in this case is an iterative process, more computational time is needed to evaluate the limit state equation. Moreover, the probabilities to be estimated are very low (of the order of 10^{-5}–10^{-7}) since high-reliability indices are required. This suggests that advanced simulation techniques have to be applied in the reliability assessment. That is why the asymptotic sampling technique described by Bucher (2009) is presented in this paper.

2.1. Limit state equation

The factor of safety, *FS*, for an embankment is defined using the following equation:

$$FS = \frac{M_{\mathrm{stab}} + z_{\mathrm{opt}}}{M_{\mathrm{driv}}}; \quad FS \ge 1 \tag{3}$$

where M_{stab} (kN m/m) and M_{driv} (kN m/m) are the stabilising and driving moments around the point of rotation, respectively. The centre of rotation is selected so that the factor of safety, *FS*, is minimized. An optimization procedure is performed to determine the location of the rotation centre in order to find the minimum *FS*.

Since both the driving moment and the stabilising moment can be calculated in different ways, the factor of safety, *FS*, is not directly a measure of the reliability of the structure. *FS* is determined using characteristic values of the load and resistance parameters combined with the given set of partial factors.

If the minimum *FS* is less than 1, the embankment is unstable and actions increasing the stabilising moment should be considered. That is why z_{opt} is introduced as a *design parameter*, modelling the required stabilising moment in order to obtain $FS \ge 1$.

The driving and stabilising moments are calculated using circular slip surfaces. z_{opt} is found as the necessary moment in order to obtain equilibrium in the deterministic design situation. The limit state equation for the railway embankment is defined by the following equation:

$$g(x) = \frac{M_{\mathrm{stab}} + z}{M_{\mathrm{driv}}} \tag{4}$$

where z is an outcome of the stochastic variable modelling the uncertainty related to the stabilising moment, z_{opt}, from the stabilising berms. All parameters in Eq. (4) are outcomes of the individual stochastic variables.

2.2. Stochastic modelling of soil

The undrained shear strength of the soft layer under the embankment is modelled as a stochastic field, taking into account the dependency of overburden stress and correlation between strengths of different positions. The mean value and standard deviation of the undrained shear strength of a given point are

$$\mu_{c_u} = 0.4H_{\mathrm{tot}}\gamma', \quad \sigma_{c_u} = V_{c_u}\mu_{c_u} \tag{5}$$

where H_{tot} (m) is the total height of the soil above the calculation point, γ' (kN/m^3) is the effective unit weight of the soil, and V_{c_u} is the coefficient of variation for the soil. If there are several layers above the calculation point, the multiplication is carried out for each layer separately, thus accounting for the strength increase of soils under the embankment due to consolidation. The mean values of the undrained shear strength c_u modelled by Eq. (5) is illustrated in Fig. 1 for an embankment with the height of 10 m.

The strength is, however, random of nature and therefore local variations occur (Nishimura et al., 2011; Wang et al., 2011). The correlation coefficient of the shear strength is modelled by the following function (JCSS, 2006):

$$\rho_{ij} = \exp\left(\frac{|x_i - x_j|}{d_x} - \frac{|y_i - y_j|}{d_y}\right) \tag{6}$$

where d_x (m) and d_y (m) are the correlation lengths in the horizontal and vertical directions, respectively. They are chosen based on literature study (JCSS, 2006), and on Danish geotechnical experts assessing borehole profiles and vane shear tests from the site (Lodahl et al., 2012). The values of d_x and d_y used in the analysis were 3 m and 1 m, respectively.

The stochastic field modelling the undrained shear strength is discretized and the discretized shear strengths are modelled by the vector, $\boldsymbol{c}_u$, which in each entry holds the undrained shear strength

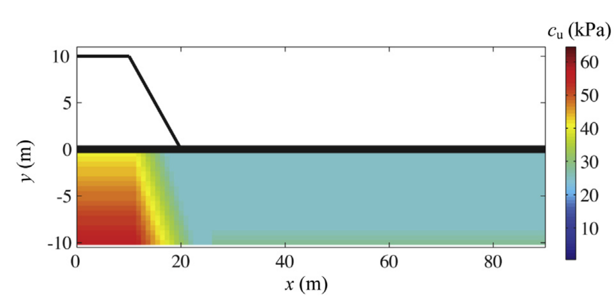

Fig. 1. Mean values of c_u of soft soil deposits under the embankment as modelled by Eq. (5).