Journal of Rock Mechanics and Geotechnical Engineering 6 (2014) 466–476

Contents lists available at ScienceDirect

Journal of Rock Mechanics and Geotechnical Engineering

journal homepage: www.rockgeotech.org

Full length article

# Theoretical and numerical studies of crack initiation and propagation in rock masses under freezing pressure and far-field stress

Yongshui Kang [a,*], Quansheng Liu [a], Xiaoyan Liu [b], Shibing Huang [a]

[a] State Key Laboratory of Geomechanics and Geotechnical Engineering, Institute of Rock and Soil Mechanics, Chinese Academy of Sciences, Wuhan, 430071, China

[b] Key Laboratory of Rock Mechanics in Hydraulic Structural Engineering of Ministry of Education, School of Civil Engineering and Architecture, Wuhan University, Wuhan, 430072, China

ARTICLE INFO

*Article history:*
Received 31 December 2013
Received in revised form
12 May 2014
Accepted 29 May 2014
Available online 9 August 2014

*Keywords:*
Freeze–thaw action
Freezing pressure
Stress intensity factor (SIF)
Crack propagation

ABSTRACT

Water-bearing rocks exposed to freezing temperature can be subjected to freeze–thaw cycles leading to crack initiation and propagation, which are the main causes of frost damage to rocks. Based on the Griffith theory of brittle fracture mechanics, the crack initiation criterion, propagation direction, and crack length under freezing pressure and far-field stress are analyzed. Furthermore, a calculation method is proposed for the stress intensity factor (SIF) of the crack tip under non-uniformly distributed freezing pressure. The formulae for the crack/fracture propagation direction and length of the wing crack under freezing pressure are obtained, and the mechanism for coalescence of adjacent cracks is investigated. In addition, the necessary conditions for different coalescence modes of cracks are studied. Using the topology theory, a new algorithm for frost crack propagation is proposed, which has the capability to define the crack growth path and identify and update the cracked elements. A model that incorporates multiple cracks is built by ANSYS and then imported into FLAC$^{3D}$. The SIFs are then calculated using a FISH procedure, and the growth path of the freezing cracks after several calculation steps is demonstrated using the new algorithm. The proposed method can be applied to rocks containing fillings such as detritus and slurry.



## 1. Introduction

Low temperatures and harsh weather can be observed in nearly half of the earth's land area in winter. Frost weathering in cold regions, where a quarter of the land mass is under permafrost or covered by snow, poses a serious threat to the stability of geotechnical engineering endeavors. Frost shattering caused by freeze–thaw cycles in moist rocks, as well as expansion and contraction due to temperature changes, is the major reason for physical weathering of rocks (Yang and Chen, 2004; Seto, 2010; Kang et al., 2012a, 2013). The freeze–thaw deterioration of rock is mainly reflected by the propagation of cracks under freezing pressure. Therefore, the water–ice phase transition and its effect on the evolution of the crack network should be investigated to reveal the mechanism of freeze–thaw damage to rocks.

The freeze–thaw action is caused by temperature fluctuations around the freezing point of water. Freeze–thaw action, which is also referred to as ice crystal growth or frost shattering, occurs when water in cracked rock is frozen and expands. As shown in Fig. 1, when water in cracks is frozen, a volume expansion of approximately 9% occurs, initiating a large freezing pressure that deepens and expands the crack width. When ice in the rock thaws, the water can flow further into the rock through these new cracks, facilitating the next freeze–thaw cycles. Thus, the repeated freeze–thaw processes of water in the cracks have long been recognized as a significant incentive for mechanical weathering of rocks (Hilbich, 2010; Su et al., 2010).

Frost damage in rocks is mainly caused by crack propagation under freezing pressure. The heterogeneous features of natural rocks give rise to various types of defects or local stress concentrations that might be the sources of microcracks. Based on the assumption that pre-existing microcracks are considered the most likely sources for crack propagation, Paterson and Wong (2005) analyzed the sliding crack model in detail.

In recent years, many studies have concentrated on the frost damage induced by freeze–thaw action in rock masses. However, as a result of the complicated freeze–thaw process and mathematical difficulties, most of the previous research concentrated on

* Corresponding author. Tel.: +86 27 87199185.
*E-mail address:* kys85@163.com (Y. Kang).

Peer review under responsibility of Institute of Rock and Soil Mechanics, Chinese Academy of Sciences.



http://dx.doi.org/10.1016/j.jrmge.2014.05.004

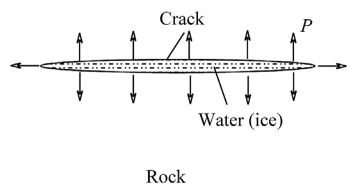


**Fig. 1.** Sketch of freezing-induced crack.

special cases, and the evolution of the crack network caused by freeze–thaw action has rarely been reported (Matsuoka, 1990, 2008; Neaupane et al., 1999; Matsuoka and Murton, 2008).

In this regard, the present paper tries to present a theoretical method to predict crack propagation under freezing pressures and far-field stresses. The direction and length of crack propagation are derived based on the Griffith theory of brittle fracture mechanics. The conditions necessary for different coalescence modes of cracks are also studied.

## 2. Stress intensity factors of freezing-induced rock cracks

### 2.1. Stress intensity factors

Rock is basically regarded as a brittle material and as such it cannot resist large tensile stresses. In general, cracks in rocks under freezing pressures and far-field stresses can be regarded as a mixed mode of tensile crack (Mode I) and shear crack (Mode II). The freezing pressure caused by volume expansion of ice can be considered as a normal pressure imposed on the crack plane, while the far-field stress might induce both normal and shear stresses on the crack plane. The sliding crack model considers the source of tensile stress concentrations located at the tips of inclined pre-existing cracks. The far-field stress would induce shear traction on the crack plane. If this shear traction is sufficiently high to overcome the frictional resistance along the interface between the crack and the ice, frictional slip occurs and a tensile stress concentration is induced at the tips of the sliding crack, with the appearance of wing cracks (Matsuoka, 2008). The driving force for nucleation is characterized by the stress intensity factor (SIF), $K_I$, at the position of wing crack initiation. With the increasing load or freezing pressure, $K_I$ will increase to the critical value $K_{IC}$ (the fracture toughness) at which point a wing crack is nucleated.

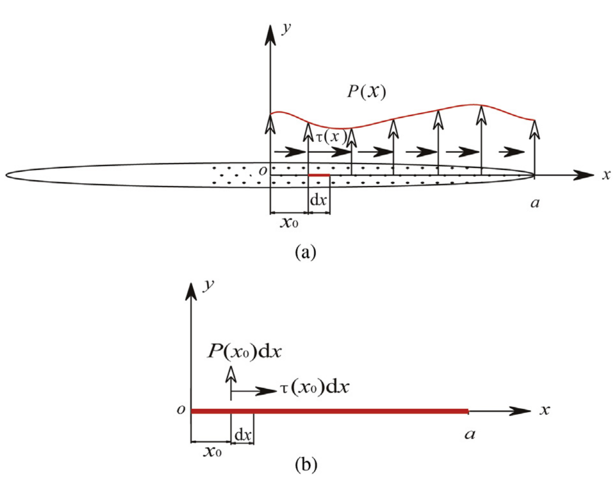


**Fig. 2.** Freezing pressure imposed on the crack. (a) Non-uniform stress. (b) Crack with concentrated force.

Numerically, the freezing pressure on the crack might not be uniform; therefore attention should be focused on the SIF induced by non-uniform stress. As shown in Fig. 2a, the model is conceived as a plane stress problem. Normal and shear stresses are non-uniformly distributed on the faces of the crack. Therefore, the integration method should be used for this problem. The non-uniform stress is roughly transformed into numerous point forces as shown in Fig. 2b, which has been studied in previous research.

The SIFs at a crack tip caused by the point force on the crack plane can be expressed as follows (Yin, 1992; China Aviation Academy, 1993):

$$dK_I = \frac{P(x)dx}{2\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}} + \frac{\tau(x)dx}{2\sqrt{\pi a}}\left(\frac{\kappa-1}{\kappa+1}\right) \tag{1a}$$

$$dK_{II} = -\frac{P(x)dx}{2\sqrt{\pi a}}\left(\frac{\kappa-1}{\kappa+1}\right) + \frac{\tau(x)dx}{2\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}} \tag{1b}$$

where $P(x)$ and $\tau(x)$ are the normal and shear forces at point $x$, respectively; $a$ is the half length of the crack; $dK_I$ and $dK_{II}$ are the Mode I and Mode II SIFs at the crack tip caused by the concentrated force. For the plane stress problem, $\kappa = (3-\nu)/(1+\nu)$, where $\nu$ is the Poisson's ratio of the rock. Thus, the total SIFs caused by the non-uniform stress can be expressed as

$$K_I = \int_{-a}^{+a}\left[\frac{P(x)}{2\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}} + \frac{\tau(x)}{2\sqrt{\pi a}}\left(\frac{\kappa-1}{\kappa+1}\right)\right]dx \tag{2a}$$

$$K_{II} = \int_{-a}^{+a}\left[-\frac{P(x)}{2\sqrt{\pi a}}\left(\frac{\kappa-1}{\kappa+1}\right) + \frac{\tau(x)}{2\sqrt{\pi a}}\sqrt{\frac{a+x}{a-x}}\right]dx \tag{2b}$$

It is necessary to define the effective SIFs when the results are applied to numerical simulations. As shown in Fig. 3, the shear and normal stresses are considered to act on elements. For the plane stress problem, the effective SIFs can be calculated as follows:

$$K_I^e \approx \sum_{i=1}^{n}\left[\frac{P_iA_i}{2\sqrt{\pi a}}\sqrt{\frac{a+x_i}{a-x_i}} + \frac{\tau_iA_i}{2\sqrt{\pi a}}\left(\frac{\kappa-1}{\kappa+1}\right)\right] \tag{3a}$$

$$K_{II}^e \approx \sum_{i=1}^{n}\left[\frac{P_iA_i}{2\sqrt{\pi a}}\left(\frac{\kappa-1}{\kappa+1}\right) + \frac{\tau_iA_i}{2\sqrt{\pi a}}\sqrt{\frac{a+x_i}{a-x_i}}\right] \tag{3b}$$

where $K_I^e$ and $K_{II}^e$ are the effective Mode I and Mode II SIFs, respectively; $P_i$ and $\tau_i$ are the normal and shear stresses acting on the element; $i$ denotes the element ID number; $A_i$ is the area of the

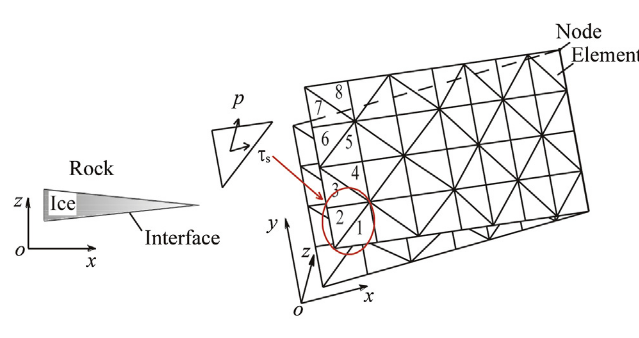


**Fig. 3.** Elements of the crack. (a) Sketch of a freezing-induced crack. (b) Elements of the crack interface.