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New artificial neural networks for true triaxial stress state analysis and demonstration of intermediate principal stress effects on intact rock strength

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ABSTRACT

Simulations are conducted using five new artificial neural networks developed herein to demonstrate and investigate the behavior of rock material under polyaxial loading. The effects of the intermediate principal stress on the intact rock strength are investigated and compared with laboratory results from the literature. To normalize differences in laboratory testing conditions, the stress state is used as the objective parameter in the artificial neural network model predictions. The variations of major principal stress of rock material with intermediate principal stress, minor principal stress and stress state are investigated. The artificial neural network simulations show that for the rock types examined, none were independent of intermediate principal stress effects. In addition, the results of the artificial neural network models, in general agreement with observations made by others, show (a) a general trend of strength increasing and reaching a peak at some intermediate stress state factor, followed by a decline in strength for most rock types; (b) a post-peak strength behavior dependent on the minor principal stress, with respect to rock type; (c) sensitivity to the stress state, and to the interaction between the stress state and uniaxial compressive strength of the test data by the artificial neural networks models (two-way analysis of variance; 95% confidence interval). Artificial neural network modeling, a self-learning approach to polyaxial stress simulation, can thus complement the commonly observed difficult task of conducting true triaxial laboratory tests, and/or other methods that attempt to improve two-dimensional (2D) failure criteria by incorporating intermediate principal stress effects.

1. Introduction

The case for the significance of the intermediate principal stress, σ_2, to rock brittle fracture and rock strength, has been historically well established (Murrell, 1963; Mogi, 1971; Takahashi and Koide, 1989; Haimson and Chang, 2000; Malama, 2001; Colmenares and Zoback, 2002; Haimson and Rudnicki, 2010). A major challenge, however, is that understandably scarce polyaxial laboratory data were obtained from “true triaxial” tests ($\sigma_2 \neq \sigma_3$, σ_3 represents minor principal stress), as opposed to conventional triaxial tests ($\sigma_2 = \sigma_3$) to buttress experimental, analytical or computer models (e.g. Kim and Lade, 1984; Christensen et al., 2004; Pan et al., 2012). Conducting true triaxial tests is not trivial, and test machinery capable of independently incorporating all three principal stresses is complicated to be designed. The shortage of data thus makes it a challenge to undertake comprehensive studies to enhance understanding of the true nature of rock failure/strength, as a means of substantiating theoretical models.

To compensate for these deficiencies, some efforts have been redirected to place emphasis on the effect of only the relationship of the major principal stress, σ_1, and the minor principal stress, σ_3, on rock strength as for example evidenced by a plethora of two-dimensional (2D) rock strength criteria in the literature. However, evidence has been accumulating that the role of the intermediate principal stress, σ_2, in rock fracture and/or rock strength can neither be trivialized nor ignored (Haimson, 2006). For example, Murrell (1963) demonstrated that Carrara marble is stronger under triaxial extension ($\sigma_2 = \sigma_1$) than under triaxial compression ($\sigma_2 = \sigma_3$), i.e. conventional triaxial tests in compression, in which the intermediate principal stress is equal to the minor principal

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stress, do not lead to a general failure criterion. Handin et al. (1967) showed that σ_2 caused the angle between the failure plane at brittle fracture and the direction of σ_1 to decrease between triaxial compression and triaxial extension in Solnhofen limestone. Wiebols and Cook (1968) developed a failure criterion, based on effective strain energy, to show that, for a constant value of σ_3, the strength as σ_2 is raised from its initial value of $\sigma_2 = \sigma_3$ to where it reaches a peak and then declines to $\sigma_2 = \sigma_1$ (Fig. 1). Mogi's experiments on carbonates and silicates (Mogi, 1971) demonstrated that the largest effect of σ_2 on strength is reached at a level well inside the range between $\sigma_2 = \sigma_3$ and $\sigma_2 = \sigma_1$. The work on sandstones and shales by Takahashi and Koide (1989) showed that these rock strengths were not only dependent on the absolute value of σ_2, but also dependent on the relative value of σ_2. On the other hand, Cai (2008) demonstrated numerically that little strength increase occurred in rock when σ_2 was increased substantially at low values of σ_3. Chang and Haimson (2000) showed that the increase in strength as a function of σ_2 for constant σ_3 is substantial, and in some cases, as much as 50% or more over the commonly used conventional triaxial strength, and that higher intermediate principal stress magnitudes appeared to extend the elastic range of the stress–strain behavior for a given σ_3, thereby retarding the onset of the failure process. Perhaps the mixed effects of the intermediate principal stress were best highlighted by Chang and Haimson (2005) who indicated that, for certain rock types (e.g. hornfels or metapelite), compressive strength σ_1 does not vary significantly regardless of the applied σ_2 after all.

Interestingly, some more recent studies (Haimson and Rudnicki, 2010; Ma and Rodriguez, 2012) proposed *symmetrical* failure envelopes with respect to Mogi's stress factor, β (Mogi, 1971), different from classical three-dimensional (3D) failure envelopes (such as those depicted in Fig. 1). The stress factor, β, defined as $\beta = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$, ranges from 0 ($\sigma_2 = \sigma_3$) to 1 ($\sigma_2 = \sigma_1$), as shown in Fig. 2. In other words, the symmetrical failure envelopes imply that the rock strength at triaxial extension ($\sigma_2 = \sigma_1$) can be equal to the strength at triaxial compression ($\sigma_2 = \sigma_3$). This observation, contradicting prevailing understanding (e.g. Drucker and Prager, 1952; Murrell, 1963; Wiebols and Cook, 1968; Lade and Duncan, 1973) that rock strength is always higher at triaxial extension than at triaxial compression, implies that the symmetry imposed by the stress factor could affect the range of applicability of some 3D rock failure criteria commonly used in rock engineering (Ma and Rodriguez, 2012).

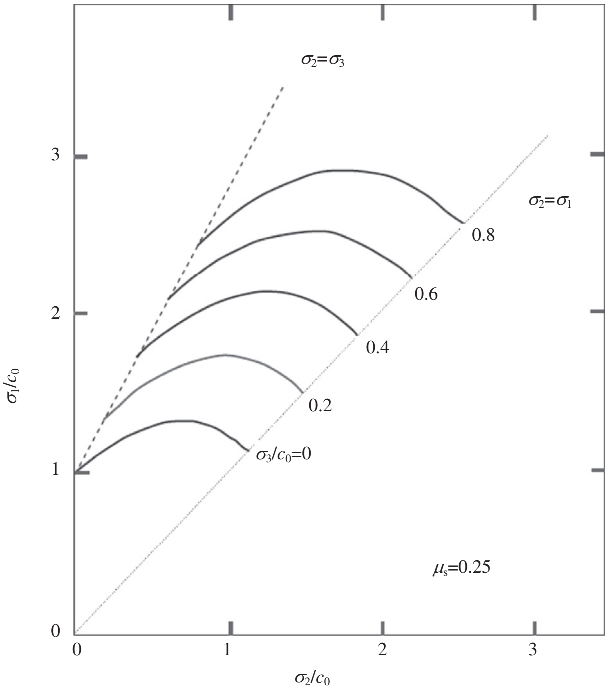

Fig. 1. The classical Wiebols and Cook curves illustrating the effects of the intermediate principal stress σ_2 on true triaxial rock strength (after Haimson, 2006). c_0 is the uniaxial compressive strength.

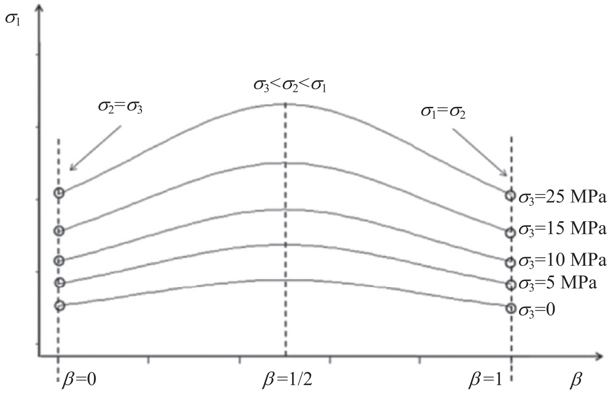

Fig. 2. Characteristic symmetry imposed by Mogi's stress factor, β, on true triaxial rock strength. Note that in this plot the rock strength at triaxial extension can be equal to the strength at triaxial compression due to symmetry (modified after Ma and Rodriguez, 2012).

In terms of mechanism, some studies attribute intermediate principal stress effects to extended evolution of localized deformation that ultimately needs significant additional strain for failure (Haimson and Rudnicki, 2010), inhomogeneous distribution of localized shear strains in shear bands with respective localized stresses (Christensen et al., 2004), and 3D interaction of microcracks prior to shear failure (Healy et al., 2006).

The challenge remains, therefore, to enhance current understanding on the effects of the intermediate principal stress on brittle/ductile rock behavior through more laboratory testing. It is therefore imperative to explore/develop new avenues to complement laboratory experiments, such as analytical approaches or computer modeling techniques (Kim and Lade, 1984; Christensen et al., 2004; Pan et al., 2012).

In summary, the main objectives of this study are:

(1) To develop new artificial neural network (ANN) models which predict stress state factors, β, from laboratory tests for several rock types. The stress state factor, β, defined above was selected as the objective parameter, because it not only allows one to normalize the influence of σ_2 (Smart et al., 1999; Alexeev et al., 2008; Zhang et al., 2010) on rock strength, but also enables one to equally treat reported laboratory tests subjected to different testing modes or stress states (Ma and Rodriguez, 2012).

(2) To show that the output from the new ANN models can act as tools to investigate/substantiate the major effects of σ_2 on rock strength discussed above, i.e.:
  (a) The characteristic that as σ_2 is raised from $\sigma_2 = \sigma_3$ to $\sigma_2 = \sigma_1$, the strength σ_1 for a constant σ_3 first increases, reaches a maximum at some intermediate value of σ_2, and then decreases to a value greater than the conventional triaxial equivalent value when $\sigma_2 = \sigma_1$.
  (b) In certain cases there is no clear trend toward an eventual decrease in strength at higher σ_2.
  (c) The observation that in certain cases a steady state is reached when the level of confining stress σ_3 is nearly equal