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Theoretical investigation of interaction between a rectangular plate and fractional viscoelastic foundation

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ABSTRACT

The interaction between plates and foundations is a typical problem encountered in geotechnical engineering. The long-term plate performance is highly dependent on the rheological characteristics of ground soil. Compared with conventional linear rheology, the fractional calculus-based theory is a more powerful mathematical tool that can address this issue. This paper proposes a fractional Merchant model (FMM) to investigate the time-dependent behavior of a simply supported rectangular plate on viscoelastic foundation. The correspondence principle involving Laplace transforms was employed to derive the closed-form solutions of plate response under uniformly distributed load. The plate deflection, bending moment, and foundation reaction calculated using the FMM were compared with the results obtained from the analogous elastic model (EM) and the standard Merchant model (SMM). It is shown that the upper and lower bound solutions of the FMM can be determined using the EM. In addition, a parametric study was performed to examine the influences of the model parameters on the time-dependent behavior of the plate–foundation interaction problem. The results indicate that a small fractional differential order corresponds to a plate resting on a sandy soil foundation, while the fractional differential order value should be increased for a clayey soil foundation. The long-term performance of a foundation plate can be accurately simulated by varying the values of the fractional differential order and the viscosity coefficient. The observations from this study reveal that the proposed fractional model has the capability to capture the variation of plate deflection over many decades of time.

1. Introduction

The interaction between a loaded plate and the soil foundation is a typical problem in foundation and pavement engineering. To solve the plate–foundation interaction problem, the well-known Winkler's foundation model is widely adopted (e.g. Matsunaga, 2000; Buczkowski and Torbacki, 2001; Huang and Thambiratnam, 2002; Zhong and Zhang, 2006). However, significant time-dependent phenomena of plates under surface loading have been observed in field, which were mainly induced by the rheological properties of ground soil. In the past few decades, the behavior of a plate resting on the viscoelastic foundation has been theoretically examined by numerous studies (e.g. Nassar, 1981; Zaman et al., 1991; Sun, 2003). In the 1950s and 1960s, the Maxwell model, the Kelvin–Voigt model, and the Merchant model are three commonly used rheological models. These simple viscoelastic models have only two or three parameters and therefore the prediction accuracy is fairly poor. More model parameters were needed to make the predictions more accurate, but difficulties in determining the parameter values arose (Chen et al., 2006).

Gemant (1936) for the first time introduced the fractional constitutive models of viscoelastic materials. In the constitutive equations of the proposed models, the integer-order differential operators were replaced by fractional-order ones. Over the past few decades, the fractional derivative viscoelastic models have shown their powerfulness in describing viscoelastic behavior of materials (Welch et al., 1999; Mainardi, 2012). Up to now, there have been a very limited number of studies that used the fractional calculus-based models to solve geotechnical problems (e.g. Atanackovic and Stankovic, 2004; Dikmen, 2005), especially the plate-

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foundation interaction problem. The main reason for this problem is the complexity involved in numerical analysis of fractional models. In the studies of Yin et al. (2007, 2013), a single fractional derivative element was proposed to describe the rheological properties of soils and rocks under different loading conditions. Zhu et al. (2011, 2012) established a fractional model by replacing the dashpot in the standard Kelvin–Voigt model with the fractional element. This model was used to analyze the ground deformation and the plate performance. One apparent drawback of this model is that it cannot account for the instantaneous deflection for a loaded plate on the viscoelastic foundation. Therefore, a more advanced fractional model with a reasonable number of parameters is necessary.

In this paper, a fractional Merchant model (FMM) is proposed to describe the time-dependent plate–foundation interaction problem. The solutions of plate deflection, bending moment, and foundation reaction are presented and compared with the calculated results of elastic and standard viscoelastic models. Through the analyses of a numerical example, the effectiveness of this four-parameter model is verified. A parametric study is then undertaken to examine the influences of the model parameters on the predicted results.

2. Fractional Merchant model (FMM)

2.1. Basics of fractional calculus

The nth derivative of a function $f(t)$ is expressed as $D^n f(t) = \mathrm{d}^n f(t)/\mathrm{d}t^n$. If n is replaced by a fraction, this expression becomes a fractional derivative. Fractional calculus is usually expressed in terms of Riemann–Liouville definition. The Riemann–Liouville fractional integration of function $f(t)$ of order v (Miller and Ross, 1993) is defined as

$$ {}_0D_t^{-v}f(t) = \frac{1}{\Gamma(v)}\int_0^t (t-\xi)^{v-1}f(\xi)\mathrm{d}\xi \quad (\mathrm{Re}(v)>0, t>0) \tag{1}$$

where the subscripts 0 and t at the left and right sides of D refer to the limits of the integration; $\Gamma(v)$ is the Gamma function with argument v. Let $[\alpha]$ be the smallest integer that exceeds α, the Riemann–Liouville fractional derivative of order α (Miller and Ross, 1993) is

$$ {}_0D_t^{\alpha}f(t) = {}_0D_t^{[\alpha]}[{}_0D_t^{-v}f(t)] \quad (\mathrm{Re}(\alpha)>0, t>0) \tag{2}$$

where $v = [\alpha] - \alpha > 0$. In the following derivation, the fractional derivative of the Riemann–Liouville type of order α is denoted as D_{RL}^{α}.

2.2. Generalization of the FMM

In the theoretical rheology, the relationships between stress $\sigma(t)$ and strain $\varepsilon(t)$ of a spring and a dashpot can be expressed in terms of differential operators:

$$\left.\begin{aligned} \sigma(t) &= ED_{RL}^{0}\varepsilon(t) \\ \varepsilon(t) &= \eta D_{RL}^{1}\varepsilon(t) \end{aligned}\right\} \tag{3}$$

where E and η are the elastic modulus and viscosity coefficient, respectively.

The fractional rheological models are on the basis of an element called "intermediate model" by Smit and de Vries (1970), or "spring-pot" by Koeller (1984). The fractional derivative element shown in

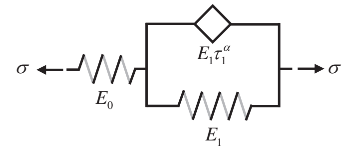

Fig. 1. Four-parameter FMM.

Fig. 1 is represented by a diamond, which has been adopted by many scholars (Bagley and Torvik, 1979; Welch et al., 1999; Dikmen, 2005). Let $\tau = \eta/E$ be the creep time, the constitutive equation of the fractional derivative element can be expressed as

$$\sigma(t) = E\tau^{\alpha}D_{RL}^{\alpha}\varepsilon(t) \quad (0 \le \alpha \le 1) \tag{4}$$

where D^{α} is the fractional differentiation defined by Eq. (2). It is noted that for $\alpha = 0$, the model defined by Eq. (4) is a spring. In the case of $\alpha = 1$, Eq. (4) can be the constitutive equation of a dashpot. The coefficient α is therefore considered to be a dimensionless parameter concerning the memory of materials (Koeller, 1984).

The Merchant model consists of a Kelvin–Voigt model and a spring connected in series. As shown in Fig. 1, if the dashpot in the Kelvin–Voigt model is replaced by a fractional derivative element, the FMM is obtained. The stress–strain relationship of this model can be expressed as

$$E_0(D_{RL}^{\alpha} + 1/\tau_1^{\alpha})\varepsilon(t) = (D_{RL}^{\alpha} + 1/t_1^{\alpha})\sigma(t) \tag{5}$$

where $\tau_1 = \eta/E_1$, $t_1 = \tau_1/\sqrt[\alpha]{1+E_0/E_1}$. It is obvious that if $\alpha = 1$, the FMM collapses to the standard Merchant model (SMM).

3. Closed-form solutions using the fractional soil–foundation interaction model

As shown in Fig. 2, a rectangular plate rests on a fractional Merchant foundation with an average thickness of d. The plate is simply supported on all four edges and is subjected to a uniformly distributed load of q_0. The length, width and thickness of this plate are a, b and h, respectively. The governing equation for plate deflection $w(x, y)$ is

$$D\nabla^2\nabla^2 w(x,y) + R(x,y) = q_0 \tag{6}$$

where $R(x, y)$ is the foundation reaction; D is the flexural rigidity of the plate defined by

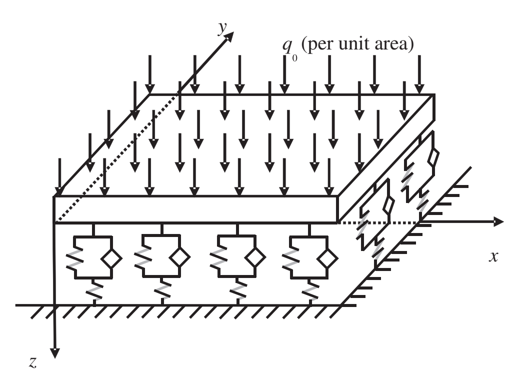

Fig. 2. Schematic illustration of a loaded rectangular plate resting on a fractional Merchant foundation.