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Effect of graph generation on slope stability analysis based on graph theory

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A B S T R A C T

Limit equilibrium method (LEM) and strength reduction method (SRM) are the most widely used methods for slope stability analysis. However, it can be noted that they both have some limitations in practical application. In the LEM, the constitutive model cannot be considered and many assumptions are needed between slices of soil/rock. The SRM requires iterative calculations and does not give the slip surface directly. A method for slope stability analysis based on the graph theory is recently developed to directly calculate the minimum safety factor and potential critical slip surface according to the stress results of numerical simulation. The method is based on current stress state and can overcome the disadvantages mentioned above in the two traditional methods. The influences of edge generation and mesh geometry on the position of slip surface and the safety factor of slope are studied, in which a new method for edge generation is proposed, and reasonable mesh size is suggested. The results of benchmark examples and a rock slope show good accuracy and efficiency of the presented method.

1. Introduction

Graph theory is an important branch of combinatorial mathematics. The theory originated from the Koenigsberg bridge problem, and the mathematician Euler used the theory to address this problem. After hundreds years of development, the graph theory has been used to solve the problems of the shortest path, network flow, dynamic planning, etc. It has been widely used in engineering fields, such as the analysis of drainage pipe network system, the optimal island distribution of smart grid, the train operation plan, and the tourism route optimization (Bondy and Murty, 1976; Wang et al., 2011).

The problem of slope stability analysis (Kim and Lee, 1997; Farias and Naylor, 1998; Sarma and Tan, 2006; Zheng et al., 2009; Guo et al., 2011; Xie et al., 2011; Zhou et al., 2011; Shen et al., 2013) can be transformed to the shortest path one in the graph theory. The directed weighted graph is firstly constructed by analyzing mesh and vertex information of the model based on numerical calculation of stress field. Then the slip surface and safety factor can be found out by the shortest path algorithm.

There have been many researchers attempting to use the graph theory to analyze the critical slip surface and the safety factor. Cherkassky et al. (1993) and Xu et al. (2007) studied the slip surface and the safety factor using the Dijkstra algorithm on the basis of finite element results. Zhou et al. (2008) used the graph theory to evaluate the stability of slope under the condition of rainfall infiltration. Zhuang et al. (2008) developed the meshless graph theory method. Bellman (1957) developed the Bellman–Ford algorithm to search for the critical slip surface of jointed rock. Fang (2007) extended the graph theory to the three-dimensional slope stability analysis, and preliminarily applied it to tunnel slope stability analysis. These methods are based on the current stress state, and can overcome the disadvantages of the conventional limit equilibrium method (LEM) which cannot consider the constitutive relation of rock/soil mass. In comparison, the strength reduction method (SRM) requires iterative calculations and cannot directly locate the slip surface (Zheng et al., 2005). It is a computationally desirable method for slope stability analysis.

The critical slip surface corresponds to the minimum safety factor of slope comprising the edges and vertices of the graph. In this approach, type of edge generation, mesh geometry and mesh density will influence the results of slope stability analysis.

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Unreasonable distribution of vertices and edges will give erroneous slip surface and safety factor. This issue has not been mentioned in the previous work. For example, in the work of Zhou et al. (2012), only the influences of physical parameters of slope material were discussed based on the graph theory.

On the basis of preliminary work (Zheng et al., 2013), the influences of the type of edge generation, mesh geometry and mesh density in the graph theory on the shape of slip surface and the safety factor are investigated in this paper. Practical suggestions are presented on how to use the graph theory for slope stability analysis effectively. The test results of benchmark examples and a rock slope show good efficiency and accuracy of the method.

2. Graph theory and slope stability analysis

The graph is a collection of vertex set $V(G)$ and an edge set $E(G)$ connecting different vertices. A line that connects two vertices is termed as an edge. The graph can be classified into directed graph and undirected graph according to whether an edge has a direction or not. In directed graph, the edge is an unidirectional path, and the pair of nodes connected by each edge is defined as a sorted couple belonging to the edge set $E(G)$. An undirected graph can be viewed as a directed graph with edges of bilateral path instead of unidirectional path.

The shortest path problem aims to find the minimum "cost" or "sum of weight" among all possible paths. As shown in Fig. 1a, there are seven vertices V_0–V_6 and twelve directed edges, the cost (weight) of each edge is shown on the edge. The shortest path problem is to search for a path that gives the minimum sum of weight from vertex V_0 to V_6. The shortest path is $V_0 \rightarrow V_2 \rightarrow V_3 \rightarrow V_5 \rightarrow V_6$ as shown in Fig. 1b.

The essential of slope stability analysis is to determine the minimum safety factor and the corresponding critical slip surface of slope. Similar to the shortest path problem between two points, determination of the critical slip surface can be converted into searching for "vertices" between starting point and end point of the slope. In this way, the safety factor of slip surface connected by these points is the minimum.

The safety factor is defined as the ratio of resistance to sliding force along the slip surface. A weight function is introduced to measure the weight (cost) of each edge when using the graph theory to determine the minimum safety factor. As shown in Fig. 2,

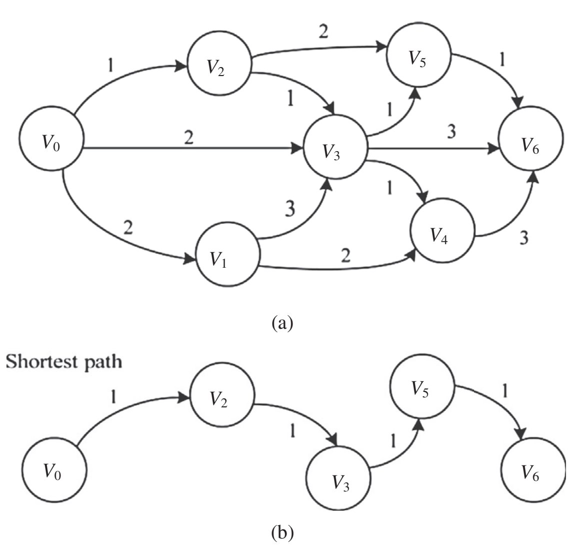

Fig. 1. Sketch of the shortest path problem.

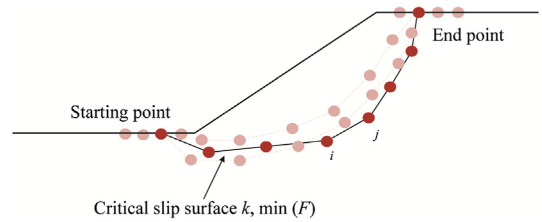

Fig. 2. Sketch of critical slip surface.

by assuming the existence of critical slip surface k, the minimum safety factor of slope can be defined as

$$F_k = \frac{\sum_i R_{k,i}}{-\sum_i S_{k,i}} \tag{1}$$

where k indicates the kth path and i indicates the ith edge comprising the kth path. Note that the summation should not be applied to k but only to i.

The minimum safety factor of all possible slip surfaces is

$$F_{\min} = \min(F_1, F_2, \cdots, F_k, \cdots, F_n) \tag{2}$$

Then, we can introduce an auxiliary function as

$$G_k = \sum G_{k,i} = \sum R_{k,i} + F_{\min} \sum S_{k,i} \tag{3}$$

and therefore the weight of each edge is simply formulated as

$$G_{k,i} = R_{k,i} + F_{\min} S_{k,i} \tag{4}$$

The problem of locating the critical slip surface can be converted into searching the corresponding path of the minimum G through the above formulas.

When using the FEM for slope stability analysis, the nodes are often used as the vertices in the graph theory, and vertices associated with that nodes are used to generate edges. Edge weights are calculated according to the stress results of the FEM. The tangential and normal forces along the edge can be obtained by averaging the stress at two vertices connected by the edge. In this paper, we use the numerical manifold method for stress analysis, and the vertices and edges in the graph theory follow the same ones as that in FEM without considering joints in the slope. For slope model containing joints, nodes of mathematical meshes and joint elements are used as the vertices in the graph theory. Thus the generation of edges in the graph theory only needs to make some changes in joint place so that the edges of the slope and the weight of edge can be obtained (Zheng et al., 2013).

For the shortest path problem, there are two basic methods, namely Dijkstra algorithm and Bellman–Ford algorithm. Other algorithms are derived on the basis of the above two methods, such as pile optimized Dijkstra algorithm, SPFA algorithm, and Floyd–Warshall algorithm, etc. Dijkstra algorithm is applicable to the shortest path problems without negative weights, while Bellman–Ford algorithm can solve the problems with negative weights. So Bellman–Ford algorithm is used to search for the critical slip surface as the edge weight of slope may be negative.

3. The influence of edge generation

The edge generation has a significant influence on the slope stability analysis. The shortest path searching algorithm can be used to find a "shortest path" that makes the minimum safety factor