Journal of Rock Mechanics and Geotechnical Engineering. 2011, 3 (Supp.): 450–456

Journal of Rock Mechanics and Geotechnical Engineering

Journal online: www.rockgeotech.org

Failure pressure calculation of fracturing well based on fluid-structure interaction

Jinzhou Zhao, Lan Ren*, Min Li, Yongming Li

State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, 610500, China

Received 14 February 2011; received in revised form 29 June 2011; accepted 12 July 2011

Abstract: Failure pressure is a key parameter in reservoir hydrofracturing operation. Existing analytical methods for calculating the failure pressure are based on the assumption that borehole fluid is under two extreme conditions: non-infiltration or complete infiltration. The assumption is not suitable for the actual infiltration process, and this will cause a great error in practical calculation. It shows that during the injection process, the dynamic variation in effective stress-dependent permeability has an influence on the infiltration, and the influence also brings about calculation errors. Based on the fluid-structure interaction and finite element method (FEM), considering partial infiltration during injection process, a numerical model for calculating rock failure pressure is established. According to the analysis of permeability test results and response-surface method, a new variation rule of rock permeability with the change of effective stress is presented, and the relationships among the permeability, confining pressure and pore pressure are proposed. There are some differences between the dynamic value of permeability-effective-stress coefficient observed herein and the one obtained by the classical theory. Combining with the numerical model and the dynamic permeability, a coupling method for calculating failure pressure is developed. Comparison of field data and calculated values obtained by various methods shows that accurate values can be obtained by the coupling method. The coupling method can be widely applied to the calculation of failure pressure of reservoirs and complex wells to achieve effective fracturing operation.

Key words: failure pressure; fluid-structure interaction; hydrofracturing; coupling method; response-surface method

1 Introduction

Calculation of failure pressure is a hot issue in the field of petroleum engineering. Hubbert and Willis [1] developed an analytical formula of failure pressure for impermeable formation without infiltration during injection process, i.e. the well-known H-W equation. Haimson and Fairhurst [2] derived an analytical formula for permeable formation with complete infiltration, i.e. the famous H-F equation. Eaton et al. [3–5] modified and improved the limitations of above two equations. The two methods consider that the behavior of wellbore fluid infiltration is under extreme conditions so that great errors in calculation are introduced. Problems caused by the errors are not very serious in the design of fracturing scheme for shallow and mid-depth layers. However, for deep or ultra-deep wells, the problems become crucial to field operation. Thus, it is necessary to carry out further study on accurate calculation methods for failure pressure.

A numerical model for failure pressure calculation was established by the authors [6]. It is a pseudo-coupling method, regarding the permeability and other parameters as constants. In fact, the permeability will change with the variation in effective formation stress during the injection process. A non-universal theoretical relationship between the permeability and effective stress [7] has been proposed. The fitting relationships among the permeability, confining pressure and pore pressure were also proposed by the authors. Meanwhile, based on the response-surface method and the core testing with gas as a fluid medium [8–10], a new rule between the permeability and the change of effective stress was presented. Therefore, a coupling model for the failure pressure calculation can be established based on the dynamic permeability and the pseudo-coupling model. The coupling model covers the infiltration of injected fluid and the interaction between the infiltration and

Doi: 10.3724/SP.J.1235.2011.00450

*Corresponding author. Tel: +86-13880716361;

E-mail: ren-ren@swpu.edu.cn

Supported by the National Natural Science Foundation of China (50774064)

reservoir stress [11–14]. Galerkin FEM is adopted as the calculation method for the coupling model [15–17]. Finally, a case is given to verify the coupling model.

2 Coupling model for rock failure pressure

The following assumptions are suggested: (1) a vertical well is located at the center of the horizontal layer; (2) the injected fluid connects with the poroelastic medium formation passing through the whole borehole (barefoot completion); (3) the fluid that flows into the formation is simulated based on the boundary condition of the well wall, and the increasing rate of the bottom pressure is a constant; (4) fluid percolation and formation stress-strain are coupled; and (5) the infiltration of the injected fluid exhibits an unsteady state. The formation permeability varies with the dynamic variation in effective stress.

2.1 Stress equilibrium equation

The constitutive relation for porous media [14] is

$$\sigma'_{ij} = \lambda \varepsilon_{\mathrm{v}} \delta_{ij} + 2G\varepsilon_{ij} \tag{1}$$

where λ is the Lame constant, G is the shear modulus of rocks, ε_{v} is the volumetric strain, ε_{ij} is the strain tensor, σ'_{ij} is the effective stress tensor, and $\delta_{ij} = 1$ $(i = j)$ or $\delta_{ij} = 0$ $(i \neq j)$.

Assuming that the strain and displacement are small, the solid skeleton is linearly elastic and related to the effective stress. According to the equilibrium of unit body stresses, the relation between the stresses is given:

$$\sigma_{ij,j} + F_i = 0 \tag{2}$$

The effective stress of the skeleton observes the Terzaghi effective stress relation [18], which can be written as

$$\sigma'_{ij} = \sigma_{ij} - \alpha p \delta_{ij} \tag{3}$$

where p is the fluid pressure in pores, σ_{ij} is the total stress tensor, and α is the Biot's coefficient.

Substituting Eq.(3) into Eq.(2), the formation stress equilibrium equation can be obtained:

$$\sigma'_{ij,j} + (\alpha p \delta_{ij})_{,j} + F_i = 0 \tag{4}$$

Substituting Eq.(1) into Eq.(4), the stress-coupled equilibrium equation in vector form is

$$\boldsymbol{S}^{\mathrm{T}}\boldsymbol{D}\boldsymbol{S}\boldsymbol{u} + \alpha\nabla\boldsymbol{p} = 0 \tag{5}$$

where $\boldsymbol{u}$ is the displacement vector; $\boldsymbol{p}$ is the fluid pressure vector in pores; and $\boldsymbol{S}$ and $\boldsymbol{D}$ can be expressed as follows:

$$\boldsymbol{S} = \begin{bmatrix} \dfrac{\partial}{\partial x} & 0 \\ 0 & \dfrac{\partial}{\partial y} \\ \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial x} \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} \lambda + 2G & \lambda & 0 \\ \lambda & \lambda + 2G & 0 \\ \lambda & 0 & G \end{bmatrix}$$

Equation (5) is the stress equilibrium equation for the coupling model, which is related to the fluid pressure. It is supposed to be solved with the continuous equation of fluid seepage.

2.2 Equation of continuity

It is assumed that the porous medium is deformable, and the fluid saturated in pores is weakly compressible, while the fluid seepage is isothermal. Meanwhile, it is assumed that the fluids percolate at a certain velocity, and the grains move at a certain speed. Accordingly, the continuous equations for fluids and solids are developed [14]:

$$\nabla \cdot (\phi \rho_{\mathrm{f}} v_{\mathrm{f}}) + \frac{\partial (\phi \rho_{\mathrm{f}})}{\partial t} = 0 \tag{6}$$

$$\nabla \cdot [(1-\phi)\rho_{\mathrm{s}} v_{\mathrm{s}}] + \frac{\partial [(1-\phi)\rho_{\mathrm{s}}]}{\partial t} = 0 \tag{7}$$

where v_{f} is the absolute velocity of fluids, v_{s} is the absolute velocity of solid grains, ρ_{f} is the fluid density, ρ_{s} is the skeleton density, ϕ is the formation porosity, and t is the time.

The positive direction of solid grain velocity is taken as the negative direction of coordinates, while the positive direction of fluid seepage velocity is taken as the positive direction of coordinates. Then, we have [7]

$$v_{\mathrm{D}} = \phi (v_{\mathrm{f}} + v_{\mathrm{s}}) \tag{8}$$

$$v_{\mathrm{D}} = -\frac{k}{\mu}\nabla p \tag{9}$$

where v_{D} is the Darcy velocity, k is the formation permeability, and μ is the fluid viscosity.

Considering Eq.(8), Eqs.(6) and (7) can be rewritten as

$$\rho_{\mathrm{f}}\nabla \cdot v_{\mathrm{D}} - \phi\rho_{\mathrm{f}}\nabla \cdot v_{\mathrm{s}} + \phi\frac{\partial \rho_{\mathrm{f}}}{\partial t} + \rho_{\mathrm{f}}\frac{\partial \phi}{\partial t} = 0 \tag{10}$$

$$(1-\phi)\rho_{\mathrm{s}}\nabla \cdot v_{\mathrm{s}} + (1-\phi)\frac{\partial \rho_{\mathrm{s}}}{\partial t} - \rho_{\mathrm{s}}\frac{\partial \phi}{\partial t} = 0 \tag{11}$$

Dividing Eqs.(10) and (11) by ρ_{f} and ρ_{s} respectively, and then combining them together, we can obtain

$$\nabla \cdot v_{\mathrm{D}} - \nabla \cdot v_{\mathrm{s}} + \frac{(1-\phi)}{\rho_{\mathrm{s}}}\frac{\partial \rho_{\mathrm{s}}}{\partial t} + \frac{\phi}{\rho_{\mathrm{f}}}\frac{\partial \rho_{\mathrm{f}}}{\partial t} = 0 \tag{12}$$