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Generation of higher harmonics in longitudinal vibration of beams with breathing cracks

D. Broda [a], L. Pieczonka [a], V. Hiwarkar [b], W.J. Staszewski [a,*], V.V. Silberschmidt [b]

[a] Department of Robotics and Mechatronics, AGH University of Science and Technology, Al. A. Mickiewicza, 30-059 Krakow, Poland
[b] Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK

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A B S T R A C T

Classical nonlinear vibration methods used for structural damage detection are often based on higher- and sub-harmonic generation. However, nonlinearities arising from sources other than damage – e.g. boundary conditions or a measurement chain – are a primary concern in these methods. This paper focuses on localisation of damage-related nonlinearities based on higher harmonic generation. Numerical and experimental investigations in longitudinal vibration of beams with breathing cracks are presented. Numerical modelling is performed using a two-dimensional finite element approach. Different crack depths, locations and boundary conditions are investigated. The results demonstrate that nonlinearities in cracked beams are particularly strong in the vicinity of damage, allowing not only for damage localisation but also for separation of crack induced nonlinearity from other sources of nonlinearities.

1. Introduction

A growing complexity of modern civil, aerospace and power-plant structures has led to development of more strict safety regulations. In order to meet these strict regulations, reliable and cost-effective maintenance methods are required. Effective maintenance not only improves safety, but also minimises the cost of ownership and mitigates unnecessary repairs. It is well known that Non-Destructive Testing (NDT) is the field of engineering that addresses this important problem, assuring the desired level of safety [1]. NDT inspections are, however, performed only at predefined time intervals and are often not sufficient to capture the evolution of damage in monitored structures. Hence, the more recent Structural Health Monitoring (SHM) approach is based on permanently mounted (bonded or embedded) networks of sensors used for continuous monitoring of structures [2,3]. Among the many available approaches to SHM, methods based on nonlinear vibration/acoustic phenomena are of special interest, gaining an increasing attention in the scientific community [4–6]. This is mainly due to the fact that the nonlinear damage detection methods are usually more sensitive to detect small damage severities than their linear counterparts [7–9].

Structural damages (e.g. cracks or delaminations) affect modal parameters, i.e. natural frequencies, damping and mode shapes of monitored structures [10,11]. Therefore, vibration-based damage detection methods monitor these parameters and relate them to possible structural damage. It is well known from theory that cracks in beams reduce stiffness locally, leading to the reduction of lower order natural frequencies. This observation leads to various open-crack formulations, in

* Corresponding author.
E-mail addresses: dbroda@agh.edu.pl (D. Broda), staszews@agh.edu.pl (W.J. Staszewski).

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which structural stiffness is decreased regardless of loading. However, in real engineering structures that are vibrated cracks remain open only when static stresses are significantly high. Otherwise cracks can easily close during compression and open during tension within one vibration cycle [11]. This opening–closing nonlinear crack effect – often referred to as *breathing crack* – has attracted many research investigations since the early work from the 1980s [12,13]. Various analytical, numerical and experimental studies have been performed including the work presented in [11,14–16]. The effect of open and closed cracks on natural frequencies of beams was investigated in [12,13,17–20]. A frequency drop was shown to be smaller for breathing – rather than for open – cracks in these investigations.

The simplest model of a *breathing crack* utilises a bi-linear stiffness relation that uses different elastic moduli for the open and closed crack:

$$m\ddot{q}+c\dot{q}+k(q)q=F(t) \tag{1}$$

where m stands for mass, c denotes damping, F is a time-dependent excitation force and stiffness $k(q)$ can be modelled as

$$k(q)=\begin{cases} k_t & \text{if } q\geq q_0 \\ k_c & \text{if } q<q_0 \end{cases} \tag{2}$$

and q denotes the crack response, q_0 is the value of the response when the crack opens or closes, k_c and k_t stand for stiffnesses for closed and open crack, respectively. More accurate physical models also involve the contact of crack edges (or faces) at non-zero velocities (contact of rough surfaces), crack-tip plasticity, friction and even temperature gradients near crack tips [21].

The analysis of higher harmonics is a possible solution to the problem of minor frequency drop related to damage [11,16,22–25]. Some of the signal processing techniques applied for higher harmonics analysis are based on the bi-spectrum [26,27], time–frequency analysis [28,29] and higher order transfer function [24]. The influence of damping on the level of nonlinearities in cracked structures and the analysis of pseudo-superharmonic resonances were investigated in [30]. Other damage indicators based on higher harmonic generation involve the application of the nonlinear output frequency response functions (NOFRFs) [31], Volterra-series response representation [32] and the multi-modal technique [33]. The latter method assumes that individual vibration modes are sensitive to different crack locations. This assumption can be used for precise location of small defects (crack depth proportional to 10 percent of specimen cross-sections). Higher harmonics in ultrasonic responses were also investigated for damage detection, e.g. in [34]. Several FE models of beam with breathing crack were presented in the literature. 2-D and 3-D frictional FE models of cracked beam were presented in [35,36]. The work involved parametric studies of the effect of crack depth, position and angle on natural frequencies of a beam. Several different configurations of cracks and positions of applied forces were investigated in [37]. The study based on a 3-D FE model of cracked beams investigated changes in natural frequencies resulting from both free and forced vibration. Not only FE, but also other techniques, like spectral elements method, were proposed for simulations of damage detection in cracked structures [38].

A 2-D FE model of beam with a breathing crack was investigated in [39–41]. The excitation frequency corresponded to harmonics and sub-harmonics of the natural frequency of the beam. Crack detection approaches based on amplitudes of higher harmonics and sub-harmonics, excursion variations and trajectory eccentricities of the phase portrait were proposed. Vibration responses measured at various locations on the beam in experimental studies demonstrated that sensor location and excitation amplitude had a negligible effect on crack detection.

It appears that nonlinear longitudinal vibration of cracked beams has attracted less research effort. Longitudinal vibration of bar with breathing cracks were investigated in a series of publications [42–44]. The dynamic compliance (receptance) is modelled in these investigations as [42]

$$L_l(l,j\omega)=\frac{l}{ES\xi}\left[\tan\xi-\frac{jX}{4\pi}\frac{\xi+\dfrac{1}{2}\sin(2\xi)}{\cos^2\xi}\right] \tag{3}$$

where $\xi=\frac{\omega l}{\gamma}$, $\gamma=(E/\rho)^{(1/2)}$, l, S and E are the length, cross-sectional area and modulus of elasticity of the bar, respectively, ω is the frequency of excitation and X stands for the absorption coefficient. This equation was transformed using a general linear theory of integral equations for straight rods, allowing for numerical simulations based on the Matlab-Simulink environment [42]. The analytical and numerical studies were validated by a series of experiments that utilised cracked aluminium samples [43]. The results of simulations demonstrated generation of higher harmonics and a decrease of natural frequencies. These nonlinear effects were dependent on crack parameters. The results also demonstrated that the intensity of higher harmonics was a function of a distance between a response location and a crack location [43]. The conclusions on the relation between these locations was rather general, and based on tests performed for simple experimental arrangements. It is also important to note that numerical simulations, demonstrating the existence of the crack localisation effect, were conducted only for a 1-D model in the Matlab Simulink environment. The term crack localisation effect refers to the local increase of a nonlinearity near the crack.

The crack localisation effect was also mentioned by other authors. Experimental studies in [45–47] indicate that the amplitude of the second harmonic is particularly strong near impact damage in composite plates. However, this effect was