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Travelling wave and non-stationary response in nonlinear vibrations of water-filled circular cylindrical shells: Experiments and simulations

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ABSTRACT

The nonlinear vibrations of a water-filled circular cylindrical shell subjected to radial harmonic excitation in the spectral neighbourhood of the lowest resonances are investigated experimentally and numerically by using a seamless aluminium sample. The experimental boundary conditions are close to simply supported edges. The presence of exact one-to-one internal resonance, giving rise to a travelling wave response around the shell circumference and non-stationary vibrations, is experimentally observed and the nonlinear response is numerically reproduced. The travelling wave is measured by means of state-of-the-art laser Doppler vibrometers applied to multiple points on the structure simultaneously. Chaos is detected in the frequency region where the travelling wave response is present. The reduced-order model is based on the Novozhilov nonlinear shell theory retaining in-plane inertia and the nonlinear equations of motion are numerically studied (i) by using a code based on arclength continuation method that allows bifurcation analysis in case of stationary vibrations, (ii) by a continuation code based on direct integration and Poincaré maps that evaluates also the maximum Lyapunov exponent in case of non-stationary vibrations. The comparison of experimental and numerical results is particularly satisfactory.

1. Introduction

The literature on nonlinear vibrations of circular cylindrical shells has been reviewed by Amabili and Païdoussis [1] and more recently by Alijani and Amabili [2]. Fundamental studies to understand the nonlinear vibrations of simply supported, thin circular cylindrical shells with and without fluid-structure interaction are those of Chen and Babcock [3], Gonçalves and Batista [4], Amabili et al. [5] and Amabili [6,7]. Experimental studies on nonlinear vibrations of circular cylindrical shells are unfortunately rare in the literature. Particularly significant experiments are reported by Chen and Bakcock [3], Chiba [8,9], Amabili et al. [10], Amabili [11], Pellicano [12] and Pellicano et al. [13]. In particular, references [3,11] present the simply supported boundary conditions.

The previous studies [3,5–7] show that the geometrically nonlinear vibrations of thin circular cylindrical shells are of softening type, for shells that are not too short or too long with respect to the radius. However, the nonlinearity is weak,

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even if it is increased if the shells contains or is immersed in water [4–6,9]. Much more significant is the nonlinear phenomenon observed in forced vibrations; this is the presence of a one-to-one internal resonance between the mode directly excited by the external harmonic force, driven mode, and the identical but orthogonal mode (companion mode) rotated by $\pi/(2n)$, where n is the number of circumferential waves. The companion mode presents nodes in correspondence of the antinodes of the driven mode. While a bit away from the resonance only the driven mode is excited by the external forcing, near the resonance both driven and companion modes became active through a phenomenon of one-to-one internal resonance that gives rise to a pitchfork bifurcation of the solution. The response of the new branch of the solution, which comes out from the pitchfork bifurcation, presents driven and companion modes which tend to have very similar vibration amplitude and to be with a phase delay of $\pi/2$, giving rise eventually to a pure travelling wave response. References [3,5–7] have shown that the travelling wave response can present Neimark–Sacker bifurcations, from which a quasi-periodic travelling wave response is started. Not much further investigation of this response has been carried out to show if this vibration can be actually chaotic even for relatively small vibration amplitudes (smaller than the shell thickness for thin shells).

Also the experiments previously reported in the literature have not permitted to completely identify the travelling wave response and its non-stationary nature. Probably the most pertinent experiments related to the present study, in which simply supported boundary conditions are considered, are those presented by Amabili [11]. However, in that case the tested stainless-steel shell was manufactured with a longitudinal seam welding and only one mode with $n=10$ circumferential waves and $m=1$ longitudinal half-wave was observed not to have a significant influence of this geometric imperfection. Even if some amplitude modulations were observed, this phenomenon was almost not investigated as well as eventual chaotic vibrations.

In the present study, the nonlinear vibrations of a water-filled circular cylindrical shell subjected to radial harmonic excitation in the spectral neighbourhood of the lowest resonances are investigated experimentally and numerically by using a seamless aluminium sample. The experimental boundary conditions are close to simply supported edges. The presence of exact one-to-one internal resonance, giving rise to a travelling wave response around the shell circumference and non-stationary vibrations, is experimentally observed and the nonlinear response is numerically reproduced. The travelling wave is measured by means of state-of-the-art laser Doppler vibrometers applied to multiple points on the structure simultaneously. Chaos is detected in the frequency region where the travelling wave response is present. The reduced-order model is based on the Novozhilov nonlinear shell theory retaining in-plane inertia and the nonlinear equations of motion are numerically studied (i) by using a code based on arclength continuation method that allows bifurcation analysis in case of stationary vibrations, (ii) by a continuation code based on direct integration and Poincaré maps that evaluates also the maximum Lyapunov exponent in case of non-stationary vibrations.

2. Reduced-order model

A circular cylindrical shell with the cylindrical coordinate system (O; x, r, θ), having the origin O at the centre of one end of the shell is considered. The displacements of an arbitrary point of coordinates (x, θ) on the middle surface of the shell are denoted by u, v and w, in the axial, circumferential and radial directions, respectively; w is taken positive outwards.

The strain-displacement relationships according to the classical Novozhilov nonlinear shell theory are used in the present study. Rotary inertia and shear deformations are neglected. The strain components ε_x, ε_θ and $\gamma_{x\theta}$ at an arbitrary point of the shell are related to the middle surface strains $\varepsilon_{x,0}$, $\varepsilon_{\theta,0}$ and $\gamma_{x\theta,0}$ and to the changes in the curvature and torsion of the middle surface k_x, k_θ and $k_{x\theta}$ by the following relationships [7]

$$\varepsilon_x = \varepsilon_{x,0} + z k_x, \tag{1a}$$

$$\varepsilon_\theta = \varepsilon_{\theta,0} + z k_\theta, \tag{1b}$$

$$\gamma_{x\theta} = \gamma_{x\theta,0} + z k_{x\theta}, \tag{1c}$$

where z is the distance of the arbitrary point of the shell from the middle surface and

$$\varepsilon_{x,0} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right], \tag{2a}$$

$$\varepsilon_{\theta,0} = \frac{\partial v}{R\,\partial\theta} + \frac{w}{R} + \frac{1}{2R^2}\left[\left(\frac{\partial u}{\partial\theta}\right)^2 + \left(\frac{\partial v}{\partial\theta} + w\right)^2 + \left(\frac{\partial w}{\partial\theta} - v\right)^2\right], \tag{2b}$$

$$\gamma_{x\theta,0} = \frac{\partial v}{\partial x} + \frac{\partial u}{R\,\partial\theta} + \frac{1}{R}\left[\frac{\partial u}{\partial x}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial\theta} + w\right) + \frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial\theta} - v\right)\right], \tag{2c}$$

$$k_x = -\frac{\partial^2 w}{\partial x^2}, \tag{2d}$$