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On the efficacy of the wavelet decomposition for high frequency vibration analyses

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ABSTRACT

This paper reports the extraordinary ability of the wavelet decomposition for vibration analyses under the framework of Rayleigh–Ritz method. Using a beam as an example, Daubechies wavelet scale functions are used as admissible functions for decomposing the flexural displacement of the structure, along with the artificial springs at the boundary, to predict vibration of an Euler–Bernoulli beam in an extremely large frequency range. It is shown that the use of wavelet basis allows reaching very high frequencies, typically covering more than 1000 modes using conventional computational facility within the available numerical dynamics of the computers with no particular care needed for round-off errors. As a side benefit, the use of spring boundary also allows handling any elastic boundary conditions through a dynamic contribution in the Hamiltonian of the beam. The wavelet decomposed approach combines the flexibility of the global methods and the accuracy of local methods by inheriting the versatility of the Rayleigh–Ritz approach and the superior fitting ability of the wavelets. Numerical results on both free and forced vibrations are given, in excellent agreement with predictions of classical methods.

1. Introduction

Vibration analyses of structures in the mid-to-high frequency range pose formidable technical changes to the existing simulation methods [1]. Particular problems arising are mainly twofold. First, wavelengths get shorter as the frequency increases, so that more structural details need to be considered in the model, leading to exorbitant computational cost. Second, due to the high sensitivity of the vibrational behavior to structural details and model parameters at higher frequencies, any inaccuracy in the model or round-off errors in numerical simulation will lead to erroneous results. In order to satisfy the demand for the vast amount of parametric and optimization studies in the preliminary design stage of structures, versatile and efficient simulation methods which can potentially cover a wide frequency spectrum is of paramount importance.

Among various so-called semi-analytical methods based on spatial discretization [2], Rayleigh–Ritz method with assumed admissible functions is probably the most popular and most studied one. It consists in approximating the unknown structural displacement through a linear combination of assumed global basis functions. The basis function should, a priori, satisfies the geometric boundary conditions, which can be tactically avoided through the use of artificial spring along

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structural boundaries [2]. Based on the Hamilton's principle, the elastic boundary conditions are apprehended in the elastic energy term in the Hamiltonian of the structures, the extremalization of which can be achieved using Lagrange's equation. There is huge amount of literature on the topic, and typical applications can be found in Cheng and Lapointe [3] and Berry et al. [4]. The properties of the basis functions significantly affect the computational efficiency, stability and accuracy of the solution. Popular candidates are polynomials [3,4] and their derivatives such as Jacobi polynomials [5], Legendre polynomials [6] or Chebyshev polynomials [7]. However, they all suffer from the ill-conditioned problem [8] at higher order. This is due to the mismatch of the high numerical dynamics of the polynomial coefficients and the limited numerical dynamics of the digital computers; which is unavoidable even for symbolic computing. It is reported that the highest polynomial order that could possibly be used is roughly 19, and 44 for symbolic case [8] for one dimensional structure. Using the rule of thumb that roughly one-third to half of the modes can be correctly predicted, only less than twenty modes can be, in principle, corrected calculated. Trigonometric functions [9] or combination of the trigonometric functions and lower-order polynomials [10,11] are also possible choice. However, trigonometric functions suffer from potential convergence problems on the boundary [11]. To the best of our knowledge, practically nothing in the open literature shows their efficiency in dealing with very high-order modes except for Beslin's work [8] in which natural modes of a plate were computed, roughly reaching what can be quantified as the mid-frequency range of the structure (880 natural frequencies within 1.5 percent of error in 2-D structure, roughly corresponding to 30 modes in 1-D scenario).

Wavelets have been widely used in signal processing in various applications including sound and vibration. However, their efficacy in vibration simulations, especially as a global basis under the Rayleigh–Ritz framework, has never been explored up to now. Inspired by the appealing features of Daubechies wavelet scale functions in numerical approximation and our recent success in acoustic simulation using Galerkin formulation [12], this paper examines the use of the Daubechies wavelet scale functions as basis functions for vibration prediction under the general framework of Rayleigh–Ritz method. Daubechies wavelet scale functions have been shown to exhibit strong ability to express any square integrable functions in finite interval, high vanishing moments and orthogonality [13]. Using a beam example, it is shown that the proposed wavelet decomposed Rayleigh–Ritz method (WDRM) allows reaching very high frequencies without being restrained by the limited numerical dynamics of the computer with no particular care for round-off errors. While inheriting all the merits of Rayleigh–Ritz method, the proposed method combines the flexibility of the global methods and the accuracy of the local methods [12]. For implementation, proper treatment should be made to evaluate the derivatives of Daubechies wavelet scale functions due to its strong oscillation and lack of closed-form expression.

It is shown in this paper that the Daubechies wavelet scale functions can be used as admissible functions in Rayleigh–Ritz method and exhibit remarkable features for the vibration analyses of beams with arbitrary elastic boundaries. The limitations of the method using conventional admissible functions can be overcome since the wavelet basis can significantly reduce the round-off errors to avoid the ill-conditioned problems generally encountered. As a result, using normal computation facility, over thousands of natural modes can be accurately predicted with an error caped at 1 percent, outperforming any other basis functions reported in the literature under the frame of Rayleigh–Ritz method. Calculation cases are chosen for demonstrating the accuracy of the proposed Wavelet-based approach. Results on natural modes (from low to high frequencies) were provided. Although individual modes are less important at extra high frequencies, they are useful parameters which allow comparisons with existing analytical solutions. We also calculate the forced vibration responses which do not require the prior calculations of the modes. Results on both free and forced vibrations allow demonstrating that the proposed method is capable of dealing with typical vibration problems in a very wide frequency range, especially at high frequencies, where most of existing methods fail.

The paper is organized as follows. Section 2 is devoted to the theoretical modeling. The formulation for the beam vibrations, detailed in Section 2.1, is based on the Hamilton's principle; the elastic boundary conditions appear through a dynamic contribution in the Hamiltonian of the beam. The extremalization of the Hamiltonian is achieved using Lagrange's equation. Numerical results are presented and discussed in Section 3. The eigen-frequency and mode shape are shown. Comparisons with predictions by classic methods are made both in free–free and pinned–pinned cases. Then the forced vibration response is presented and compared with analytical solution for a large frequency band involving thousands of modes under pinned–pinned boundary conditions.

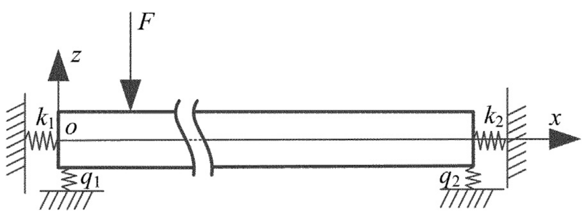

Fig. 1. Euler–Bernoulli beam model.