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The importance of the unsteady Kutta condition when modelling gust–aerofoil interaction

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ABSTRACT

The Kutta condition is applied to aerofoils with sharp trailing edges to allow for viscous effects to be considered within a simplified system of equations that are inviscid. This paper discusses in detail the inclusion of an unsteady Kutta condition at a sharp trailing edge during gust–aerofoil interaction and illustrates how the analytic solution for the far-field noise generated by this interaction changes if the unsteady Kutta condition is neglected, or more precisely, if the unsteady pressure is permitted to be singular at the trailing edge. The analytic solution, both with and without the unsteady Kutta condition, is compared with numerical results that have no imposed unsteady Kutta condition. Importantly the results agree well only when the unsteady Kutta condition is neglected in the analytic solution. This paper highlights where the far-field acoustics are most affected by neglecting the unsteady Kutta condition for a variety of singularities that can occur in the unsteady pressure at the trailing edge and shows that results permitting different behaviour in the unsteady surface pressure at the trailing edge could give significantly different far-field noise predictions, even though the surface pressure elsewhere along the aerofoil surface agrees with benchmark solutions.

1. Introduction

The unsteady Kutta condition is a well-known boundary condition applied at the trailing edge of aerofoils when investigating acoustic flow interactions (see [9] for a review). In the context of this paper we take it to enforce that the pressure jump across the trailing edge of the aerofoil is zero, and singular velocities and pressures are not permitted at the trailing edge. It allows potential-flow theory to be used in preference to the full Navier–Stokes equations for high Reynolds number interactions, which is particularly beneficial for obtaining analytical solutions to unsteady interaction problems. For the case of the full Navier–Stokes equations, the Kutta condition is thought to be valid in laminar boundary layers when unsteady (non-dimensionalised) frequencies are less than $O(Re^{1/4})$ [9] where Re is the Reynolds number based on the chord length. In this paper we are concerned with potential-flow interactions, in particular the noise generated by gust–aerofoil interaction in steady flow (uniform far upstream), which has important applications to rotor–stator interaction noise within turbo-machinery [27].

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Analytically the unsteady Kutta condition is simple to impose, along with the zero-normal velocity on the solid aerofoil surface. For flat plates in uniform steady flow at zero angle of attack with an unsteady incident gust, [19,3] discuss the analytic gust response function and the effect of the unsteady Kutta condition at the trailing edge. In particular, [3] quotes "for inviscid flow the Kutta condition is satisfied automatically as the gust passes the trailing edge". It is however less clear exactly what effect the Kutta condition has on aerofoils with non-zero thickness or camber, and this is a key problem in order to accurately predict gust–aerofoil interactions in realistic situations. Also less clear is the most appropriate way to impose the Kutta condition numerically and if it is required in real-geometry situations; a variety of different techniques exist to impose the condition [6,8,10,21], and we draw particular attention to [8, Fig. 2] which compares two different numerical Kutta conditions, which yield different surface pressure results at the trailing edge. A number of numerical methods do not impose the condition [1,15,17,22,30], and instead only a zero-normal velocity condition is imposed.

The unsteady Kutta condition acts to control the unsteady velocities and pressures at the trailing edge, therefore in this paper we pay particular attention to the surface pressure at the trailing edge. Various numerical results that do not impose an unsteady Kutta condition show singular or "spiked" trailing-edge pressures even though the pressure along the remaining aerofoil chord agrees with benchmark solutions [1,17,22,30]. Only a few of these papers go on to compare far-field noise predictions with previous results [30,17], and we see in these far-field results discrepancy between different codes that do or do not have the unsteady Kutta condition imposed. Whilst it has been reported in [28,29] that the effects of the unsteady Kutta condition on the far-field noise can be neglected, we highlight that this is only shown for flat plates, and not for aerofoils with realistic thickness or camber (such as the cases in [30,17]), and indeed [3] predicts that the unsteady Kutta condition is not a necessity for flat plates as it is self-imposed by the solution. It has been shown analytically [32] and numerically [13] that introducing non-zero thickness has a significant effect on sound generation for aerofoils of finite chord length, therefore one should be cautious about classifying zero-thickness and non-zero-thickness aerofoils in the same way.

We therefore investigate the effect of the unsteady Kutta condition on the sound generated by gust–aerofoil interaction, for aerofoils with non-zero thickness, analytically. We use this to determine whether singularities or spikes appearing in numerical surface pressure results (due to the lack of an unsteady Kutta condition) can affect the far-field acoustic results. We do so by considering one particular numerical solution in detail, first presented in [13], in which no unsteady Kutta condition is enforced and therefore there is a singularity in the unsteady surface pressure at the trailing edge.

We discuss the analytical solution for gust–aerofoil interaction in Section 2, and concentrate on the Wiener–Hopf solution for the trailing-edge interaction in Section 3, highlighting where the unsteady Kutta condition is imposed, and how the solution changes if we neglect it. We shall find that some levels of singularity in the pressure jump across the trailing edge do not alter the far-field acoustics, while others can have significant effects. Section 4 reviews the numerical method from [13], and Section 5 contains results illustrating the effect of neglecting the unsteady Kutta condition on the far-field acoustic pressure, along with a comparison of the analytical solution with the numerical results in which the unsteady Kutta condition is not imposed. Note the steady Kutta condition is imposed in the numerical result. Section 6 contains concluding remarks.

2. Analytical solution for gust–aerofoil interaction

Gust–aerofoil interaction noise, and more generally leading-edge noise, is a popular problem to study numerically [17,13], experimentally [12,11] and analytically [2,25,32]. The analytical solution for gust–aerofoil interaction in uniform flow from [25] considers only zero-thickness aerofoils, whilst the solution from [32] considers only symmetric aerofoils with non-zero thickness. In this section we present a brief overview of the solution for an aerofoil with small but non-zero thickness, camber and angle of attack (however later we simplify for a zero-camber case to illustrate the effects of the unsteady Kutta condition on non-zero thickness aerofoils). These small parameters scale with $\epsilon \ll 1$, where lengths are non-dimensionalised with respect to the aerofoil semi-chord length, b^*. Specifically we focus on high-frequency incident gusts, with reduced frequency $2\pi b^*(\lambda^*)^{-1} = k \gg 1$ (where λ^* is the wavelength of the gust) and impose $\epsilon k = O(1)$, in line with [25,32]. Further details of the model can be found in [5,4].

We study the interaction using rapid distortion theory [14]. The governing equation for the modified unsteady velocity potential, h, generated when a gust, of the form $(A_t, A_n, A_3)\mathrm{e}^{\mathrm{i}k(k_t\phi + k_n\psi + k_3 z - k_t t)}$ far upstream, interacts with an aerofoil in uniform flow with Mach number M_∞ is given by

$$\frac{\partial^2 h}{\partial \phi^2} + \frac{\partial^2 h}{\partial \psi^2} + k^2 w^2\left(1 - 2\beta_\infty^2 \epsilon q\right)h + \frac{(\gamma+1)M_\infty^4 \epsilon q}{\beta_\infty^2}\left(\frac{\partial^2 h}{\partial \psi^2} + 2\mathrm{i}k\delta\frac{\partial h}{\partial \phi} + k^2\left(w^2 + \delta^2\right)h\right)$$
$$-\frac{(\gamma+1)M_\infty^4 \epsilon}{\beta_\infty^2}\frac{\partial q}{\partial \phi}\left(\frac{\partial h}{\partial \phi} - \mathrm{i}k\delta h\right) = k\epsilon S(\phi,\psi)\mathrm{e}^{\mathrm{i}k\Omega}, \tag{1a}$$

where

$$\delta = k_t/\beta_\infty^2, \quad w^2 = (M_\infty\delta)^2 - (k_3/\beta_\infty)^2, \quad \Omega = \delta\phi + k_n\psi + \epsilon g(\phi,\psi), \tag{1b}$$