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A new nonlinear model for analyzing the behaviour of carbon nanotube-based resonators

Hamed Farokhi, Michael P. Païdoussis *, Arun K. Misra

Department of Mechanical Engineering, McGill University, Montreal, Québec, Canada H3A 0C3

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ABSTRACT

The present study develops a new size-dependent nonlinear model for the analysis of the behaviour of carbon nanotube-based resonators. In particular, based on modified couple stress theory, the fully nonlinear equations of motion of the carbon nanotube-based resonator are derived using Hamilton's principle, taking into account both the longitudinal and transverse displacements. Molecular dynamics simulation is then performed in order to verify the validity of the developed size-dependent continuum model at the nano scale. The nonlinear partial differential equations of motion of the system are discretized by means of the Galerkin technique, resulting in a high-dimensional reduced-order model of the system. The pseudo-arclength continuation technique is employed to examine the nonlinear resonant behaviour of the carbon nanotube-based resonator. A new universal pull-in formula is also developed for predicting the occurrence of the static pull-in and validated using numerical simulations.

1. Introduction

1.1. Applications and literature review

Recent technological advances have facilitated the development of nanoelectromechanical systems (NEMS), which have received considerable attention from researchers around the world, due to their unique features and widespread applications. Due to their small mass and size, NEMS devices operate at significantly high frequencies with very low power consumption; moreover, they have high quality factors [1,2] which allows for high-resolution frequency output. Such properties make NEMS devices the perfect candidate for resonant sensors which can be utilized in ultra-sensitive mass, force, and bio-sensing applications [3]. More specifically, nanomechanical resonators allow for label-free sensing and detection of biological molecules, and hence can be used as lab-on-chip biosensors for early diagnosis of different diseases, such as cancer.

The performance of a nanomechanical resonator is directly related to its dynamical behaviour. Hence, a comprehensive understanding of the dynamics of nanomechanical resonators is essential for enhancement of the detection sensitivity and development of new nano-scale resonators. As we know, one of the effective methods toward increasing the sensitivity of a resonator is to reduce its mass. In case of a nanomechanical resonator, carbon nanotubes [4] are one of the most suitable candidates due to their light mass, high stiffness, and small cross-sectional area; hence, this study will focus on analyzing the nonlinear behaviour of carbon nanotube-based (CNT-based) resonators.

* Corresponding author.
E-mail address: michael.paidoussis@mcgill.ca (M.P. Païdoussis).

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The investigations on the static and dynamic characteristics of CNT-based resonators are based on either continuum or molecular models. There are a few studies utilizing molecular dynamics (MD) simulations to predict the behaviour of carbon nanotube resonators, for instance, by Yoon et al. [5], who examined the fundamental frequency shift of a cantilevered carbon nanotube resonator encapsulating a finite nanoparticle, Kang and Kwon [6], who analyzed the resonance frequencies of a tunable carbon nanotube resonator, and Lee and Kang [7], who investigated the vibrational behaviour of cantilevered carbon nanotube resonators including an attached nano-cluster.

Reviewing the continuum-based studies, Sapmaz et al. [8] performed a theoretical investigation on the nano-electro-mechanical effects in a doubly clamped suspended carbon nanotube; they showed that the applied gate voltage bends the tube, changes the stress, and hence affects the electrical and mechanical properties. Prabhakar et al. [9] examined the frequency shifts in nanomechanical resonators; they presented a two-dimensional analysis of frequency shifts due to the thermoelastic coupling in a beam undergoing flexural vibrations. Poot et al. [10] studied the bending mode vibration in suspended carbon nanotubes; they developed a model for flexural oscillations of suspended nanotubes and presented a detailed analysis of the electrostatic force, the scaling behaviour of the model, and the gate tuning. These investigations were continued by Ouakad and Younis [11] who examined the nonlinear dynamics of electrically actuated carbon nanotube resonators under DC and AC electric loads employing an Euler–Bernoulli beam model; they used a single-mode Galerkin approximation along with a shooting technique in order to solve the reduced-order model numerically for the static and dynamic cases; however, they were unable to examine the response of the system when the slope of the frequency-response curve approached infinity and the Floquet multipliers approached unity. Hajnayeb and Khadem [12] contributed to the field by analyzing the nonlinear forced oscillations of a double-walled clamped–clamped carbon nanotube under electrostatic actuation, taking into account the interlayer van der Waals forces and mid-plane stretching. They approximated the electrostatic load via a Taylor series expansion and examined the primary and secondary resonances of the system using the method of multiple scales, based on a single-mode Galerkin approximation.

Several investigations have been carried out regarding the applications of the nano-resonators in mass detection. For instance, Li and Chou [3] analyzed the mass detection sensitivity of a carbon nanotube-based nanomechanical resonator for both cantilevered and bridged carbon nanotubes; they obtained a logarithmic linear relationship between the resonant frequency and the attached mass. Wu et al. [13] performed a linear investigation on the resonant frequency and mode shapes of a single-walled carbon nanotube (SWCNT) analytically and via continuum-based finite element method (FEM) simulations; they studied the resonant frequency shift of a fixed-free SWCNT caused by the addition of a nano-scale particle to the beam tip in order to examine the suitability of the SWCNT as a mass detector device. Further investigations were conducted by Vignola and Judge [14], who discussed the architectural considerations of micro- and nano-resonators for mass detection in different environments, such as air and water. A model for a doubly clamped carbon nanotube resonator was developed by Mei and Li [15] in order to investigate the tunability of its resonant frequency under heating; they proposed this model to adjust the mass detection sensitivity of the resonator while it is used as a mass sensor. In another

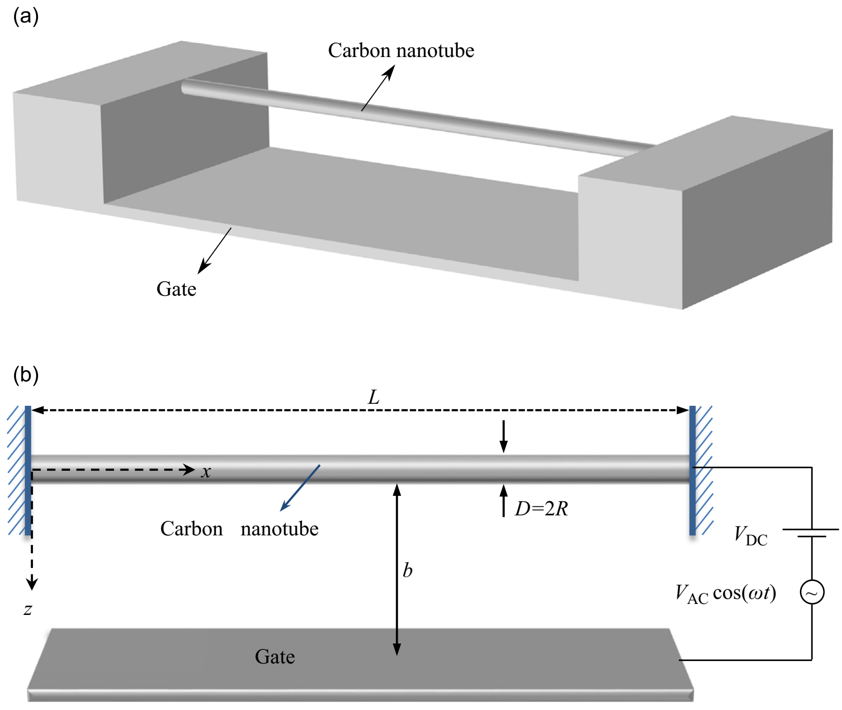

Fig. 1. (a) 3D schematic of a CNT-based resonator. (b) A simplified schematic of a CNT-based resonator showing the system parameters.