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Hysterically damped free and forced vibrations of axial and torsional bars by a closed form exact method

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ABSTRACT

Hysterically damped free and forced vibrations of axial and torsional bars are investigated using a closed form exact method. The method is exact and yields closed form expressions for the vibratory displacements. This is in contrast with the well known eigenfunction superposition method which requires expressing the distributed forcing functions and the displacement response functions as infinite sums of free vibration eigenfunctions. The hysterically damped free vibration frequencies and corresponding damped mode shapes are calculated and plotted instead of undamped free vibration and mode shapes which is typically computed and applied in vibration problems. The hysterically damped natural frequency equations are exactly derived. Accurate axial or torsional amplitude *vs.* forcing frequency curves showing the forced response due to distributed loading are displayed with various hysteretic damping parameters.

1. Introduction

As the bar executes its free vibrations, it will be seen that the amplitude decreases, even though viscous or aerodynamic damping forces are not present. The decay in amplitude is due to hysteretic damping. Hysteretic damping may be incorporated into the problem by regarding the elastic moduli as complex quantities [1], that is,

$$E^* = E(1 + \mathrm{i}\eta), \quad G^* = G(1 + \mathrm{i}\eta) \tag{1}$$

where E=Young's modulus; G=Shear modulus; $\mathrm{i} = \sqrt{-1}$; and η=material loss factor. Values of η are typically very small for metals ($10^{-6} < \eta < 10^{-3}$), but can be quite large for other materials such as rubber or plastic ($10^{-2} < \eta < 1$).

It is certain that the hysteretic damping has an effect on the natural frequencies and corresponding mode shapes. Undamped natural frequencies and corresponding mode shapes are typically computed and applied in the vibration analysis, however in the present paper hysterically damped natural frequencies and corresponding damped mode shapes are computed. Chen et al. [2] investigated free vibrations of a single degree of freedom (SDOF) system with hysteretic damping. Ribeiro et al. [3] studied free and forced vibrations of a SDOF with hysteretic damping and viscous damping. Leissa and Qatu [4] introduced the damped free vibrations of strings and bars.

Even though for the last two decades some researchers have investigated axial vibrations [5–9] and torsional vibrations [9–13] of bars, studies on forced vibrations [7,10] are very limited compared to the free vibration. Also only one paper dealt with both the axial and torsional vibration [9].

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One of the representative methods solving the forced vibration is the eigenfunction superposition method (also called "modal expansion" method). For continuous systems the standards texts often do not consider forced vibrations at all. If they do, they may lay out the continuous function form of the eigenfunction superposition (ES) method, but with very few exceptions [4,14], they exhibit no numerical results. Only a few of texts [4,15] display accurate curves showing the forced response due to distributed loading. Some show sketches of amplitude–frequency curves which are roughly drawn and not from computed data. To utilize the ES method one must first solve the free, undamped vibration problem to obtain the eigenvalues (frequencies) and corresponding eigenfunctions (mode shapes). The distributed loading must then be expanded into an infinite series, each term of which has the form of an eigenfunction. This involves evaluating integrals which may be quite complicated. The vibratory displacement is also assumed as an infinite series of eigenfunctions, each of which behaves as a single degree of freedom responding to its corresponding loading function. The displacement at any point is finally evaluated by summing the responses of the individual modes.

The present paper demonstrates a method for solving free and forced vibrations axial and torsional bars with hysteretic damping which does not require knowledge of the free vibration eigenfunctions. Rather, the problems are solved as boundary value problems in a straightforward manner, yielding closed form (instead of infinite series), exact solutions. Although the closed form exact method is straightforward, its use is less either in textbooks or in research journals when damping is considered. Leissa and Qatu [4] and Leissa [16] studied closed form exact solutions for the steady state vibrations of continuous systems subjected to distributed exciting forces with and without damping. However, they did not give numerical results. Leissa [16] studied closed form solutions for the steady state vibrations of continuous systems subjected to distributed exciting forces with and without damping. He disclosed the superiority in accuracy and efficiency of the closed form method compared with the eigenfunction superposition method. Virgin and Plaut [17] presented the effect of axial load on forced vibrations of beams with viscous damping subjected to distributed loads in a closed form exact manner.

In the present study, hysterically damped free and forced vibrations of axial and torsional bars are investigated using a closed form exact method. Instead of undamped natural frequencies and corresponding mode shapes which are typically computed and applied in vibration problems, hysterically damped natural frequencies and corresponding damped mode shapes are computed. The hysterically damped natural frequency equations are exactly derived. In the hysterically damped free vibration, effects of hysteretic damping on natural frequencies and mode shapes are studied. Some hysterically damped axial and torsional mode shapes are plotted. Accurate axial or torsional amplitude *vs.* forcing frequency curves showing the forced response due to distributed loading are displayed with various hysteretic damping parameters.

2. Hysterically damped free vibrations of axial bars

The equation of motion for hysterically damped free vibrations of an axial bar is given by

$$E(1+\mathrm{i}\eta)\frac{\partial^2 u(x,t)}{\partial x^2}=\rho\frac{\partial^2 u(x,t)}{\partial t^2} \tag{2}$$

where x=axial coordinate; $u(x,t)$ is the axial displacement in the x-direction; t=time; and ρ=mass per unit volume. The bar is assumed to be homogeneous and isotropic, thus E and ρ remain constant. On the assumption of a harmonic time response for hysterically damped axial free vibration

$$u(x,t)=U(x)\,\mathrm{e}^{\mathrm{i}t\omega_d} \tag{3}$$

where ω_d = hysterically damped axial natural frequency and $U(x)$=normal function of $u(x,t)$. Substituting Eq. (3) into the governing Eq. (2) results in

$$E(1+\mathrm{i}\eta)\frac{d^2U(x)}{dx^2}+\rho\omega_d^2U(x)=0 \tag{4}$$

Using the non-dimensional axial coordinate $\xi=x/L$, where L is a length of the bar, yields

$$(1+\mathrm{i}\eta)\frac{d^2U(\xi)}{d\xi^2}+\lambda_d^2U(\xi)=0 \tag{5}$$

where λ_d is the non-dimensional hysterically damped axial natural frequency defined by

$$\lambda_d=\omega_dL\sqrt{\frac{\rho}{E}} \tag{6}$$

General solution to Eq. (5) could be expressed by

$$U(\xi)=C_1\mathbf{e}^{\frac{\mathrm{i}\lambda_d}{\sqrt{1+\mathrm{i}\eta}}\xi}+C_2\mathbf{e}^{\frac{-\mathrm{i}\lambda_d}{\sqrt{1+\mathrm{i}\eta}}\xi} \tag{7}$$

where C_1 and C_2 are arbitrary integration constants.