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Non-stationary random vibration analysis of multi degree systems using auto-covariance orthogonal decomposition

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ABSTRACT

An algorithm that integrates Karhunen–Loeve expansion (KLE) and finite element method (FEM) is proposed to carry out random vibration analysis of complex dynamic systems excited by stationary or non-stationary random processes. In KLE, the auto-covariance function of random process is discretized using orthogonal basis functions. During the response calculations, the eigenvectors of KLE are applied as forcing functions. Three methods are proposed to carry out the random vibration analysis termed as, ***Method 1A***, ***Method 1B*** and ***Method 2***. In ***Method 1A*** and ***Method 1B***, the basis functions are chosen such that they include multiples of complete or half-cosine and sine functions over the selected time. In ***Method 2***, the basis functions are chosen to be simple piecewise constants. The proposed algorithm is applied to a 2DOF system, a cantilever beam and a stiffened panel for both stationary and non-stationary excitations. Results show that three methods can describe the statistics of the dynamic response with sufficient accuracy. However, ***Method 1A*** results have a relatively larger error than that for ***Method 1B*** and ***Method 2*** during initial transient time. The ***Method 2*** results have an excellent agreement with analytical results. Moreover, the runtime of ***Method 2*** algorithm is significantly less than both ***Method 1A*** and ***Method 1B*** algorithms even though its usage results in an increase in the number of KLE terms. Furthermore, ***Method 2***, unlike ***Method 1A*** and ***1B***, neither yields negative and/or infinite eigenvalues for the auto-covariance function nor large inaccuracies.

1. Introduction

Many engineering structures are subjected to random dynamic loads which lacks pattern and regularity in time and space domains. Random excitations often occur in many real-life vibration problems, e.g. gust loads on wings, excitations caused by turbulent boundary layers on panels, and non-stationary wind and seismic loads on tall buildings. Hence, random vibration analysis has been drawing increased attention in recent years from the designers of many aerospace, civil, and

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mechanical structures. The random processes are often simplified to be a stationary Gaussian process so that it is convenient to carry out the random analysis. However, many engineering structures encounter non-stationary and non-Gaussian random loadings. The development of the treatment of non-stationary non-Gaussian excitations is however inhibited by both extensive computational cost and mathematical intricacies. With the relentless progress in high-performance computing, numerical methods are being increasingly used to perform random vibration analysis of complex structures [1].

A random process can be described using time or frequency domain methods using auto-covariance function and power spectral density (PSD) function, respectively. For a stationary Gaussian process, the PSD function is usually used to represent the random process [2–4]. The PSD of the response under stationary Gaussian excitations can be obtained from the system transfer functions in the frequency domain, an approach that is rather mature. For the non-stationary processes, the marginal probability density functions (PDF) of the excitation may have positive real domain which might be finitely bounded. So, the auto-covariance functions are used to define the random processes in the time domain. However, the response of dynamic systems excited by non-stationary random processes has not been studied in great depth [5]. The response of dynamic systems under non-stationary random excitations [6–8] has been conducted using an integral method, meaning that integrations are performed by using analytical or numerical methods for the response calculations. This is possible for simple structures, but for large complex structures, the computational cost is too high to perform the vibration analysis. Mostly, the non-stationary excitations are decomposed into a stationary random process and a modulating function [8], which is generally limited to finite DOF systems [9].

In most applications of random analysis, it is necessary to represent the continuous random processes in terms of random variables and it can be achieved by discretizing the random processes. So, when auto-covariance functions are used to define random excitations, the first step is to discretize the auto-covariance functions while calculating the response of the system [10]. Recently, efforts have been made to obtain the random system response by using Karhunen–Loeve expansion (KLE) of the auto-covariance. The random processes can be decomposed into random variables named as basis variables and deterministic orthogonal functions in the KLE. The set of basis variables is a set of independent identically distributed random variables with zero mean and unit variance. The KLE serves as a useful and efficient tool for discretizing second-order random process with known covariance function [11]. However, the efficiency of KLE for simulating random process highly depends on the availability of accurate eigenvalues and eigenfunctions of the auto-covariance function when solving a Fredholm equation of the second kind [12]. The KLE is often carried out by using Monte-Carlo method [13] and polynomial chaos method [14] for the non-Gaussian non-stationary processes. Recently, an algorithm for decomposing auto-covariance function using orthogonal decomposition is presented by Mulani et al. [5,15] and Phoon et al. [16]. In this method, the eigenvalues and eigenfunctions are obtained by using numerical methods such as analytical method [5], Galerkin projection method [17] and collocation technique [18]. The analytical method is only efficient for simple definitions of the auto-covariance functions. The Galerkin projection method is an efficient method for solving eigenvalues and eigenfunctions of auto-covariance. However, it often yields negative eigenvalues of the auto-covariance function (based upon type of basis functions) and it also may result in numerical inaccuracies during integration while solving the Fredholm equation of the second kind. The collocation technique is another efficient method to obtain the eigenvalues and eigenvectors of the auto-covariance matrix by converting the auto-covariance function into an auto-covariance matrix. The collocation technique is not as accurate as the Galerkin projection method, although the method does not yield negative and infinite eigenvalues [5]. A wavelet-Galerkin scheme method [12] is also proposed recently, which can reduce the computational cost. However, this method does not have any clear advantage for strongly correlated systems but also ignores the orthogonality of the eigenvectors for non-stationary processes.

In the authors' previous works [5,19,20], a set of basis functions which are trigonometric basis functions and piecewise linear interpolation functions have been chosen to obtain the eigenvalues and eigenvectors for KL expansion. It has made good progress in solving simple single-degree-of-freedom system, 2DOF system and a simple beam subjected stationary and non-stationary excitations. However, the accuracy, effectiveness and computational efficiency were not deeply studied when piecewise linear basis functions were used to carry out the stationary and non-stationary random vibration analysis. Meanwhile, the application of KLE for random vibration analysis as developed by the authors was still not feasible for more complex structures. Therefore, it is necessary to further develop an approach to study the random vibration analysis of large and complex structures under stationary and non-stationary excitations.

In order to obtain a method that is suitable for complex structures' random vibration analysis, the work conducted by the authors [5,19,20] is extended here. An algorithm that integrates KLE and FEM is proposed to conduct the random vibration analysis for any dynamic system, simple or complex; and for any random excitations, stationary or non-stationary. Then, the proposed algorithm is applied to a simple 2DOF system, a continuous beam structure and a complex stiffened panel for both stationary and non-stationary excitations. The outline of this work is as follows: In Section 2, the theory of Karhunen–Loeve expansion is addressed firstly. Then, an algorithm is proposed for random response calculation of the complex structures under stationary and non-stationary excitations. The proposed method is applied from 2DOF through beam and finally extended to a stiffened panel in Section 3. Conclusions will be drawn in the last Section.

2. Karhunen–Loeve expansion

In many engineering problems, excitations are described as random processes but they are also functions of time or spatial dimensions. In this case, it is necessary to consider joint probability density functions for the excitations. This description becomes cumbersome during calculation of responses using the joint distribution of many random variables. Hence, it is advantageous to study the simple interaction of two random variables and extract as much information as