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A Generalized Corcos model for modelling turbulent boundary layer wall pressure fluctuations

A. Caiazzo *, R. D'Amico, W. Desmet

KU Leuven, Department of Mechanical Engineering, Celestijnenlaan 300B, 3001 Heverlee, Belgium

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ABSTRACT

A turbulent boundary layer (TBL) can be an important source of noise and vibration and its simulation is still an open research challenge. The stochastic pressure distribution associated with turbulence can significantly excite a structure that radiates acoustic power. In such a situation, a good description of the wall pressure field is necessary for an accurate prediction of the vibration and noise propagation. In order to tackle this issue, many TBL models have been developed since the 50s. Among others, the Corcos model has been widely used, especially because of its advantageous mathematical features. However, a major drawback is the small rate of decay for wavenumbers below the coincidence frequency.

This paper presents a novel Generalized Corcos model that allows controlling the decay in the wavenumber domain below the convective peak, yet preserving similar mathematical advantages. Such a model is built on a two-dimensional Butterworth filter, whose orders allow to modify the shape of the TBL and possibly to adapt it to different flow configurations. The main aim of this work is to compare and position the Generalized Corcos model with respect to the existing models and to highlight its possible applications.

1. Introduction

Over the past years, the aerodynamic noise generated by a turbulent boundary layer (TBL) has received a considerable amount of attention, in particular in a context of noise and vibration assessment of vehicles. When a vehicle moves in a fluid at a sufficient speed, a boundary layer develops and grows in the fluid near the structure. In such a situation, eddies correlated over a limited region cause fluctuating pressures and shear stresses on the structural surface. The wall pressure fluctuations result in flow-induced loads that can either radiate noise directly or excite the underlying structure generating vibration and noise. In many cases, this excitation is an important contributor to the interior noise of vehicles, such as aircraft, trains, ground vehicles, ships, and submarines. To tackle this problem, engineers need numerical tools and models to account for turbulent excitations early in the design process.

In a wide variety of experimental situations, it has been found that the flow near boundaries can be modelled as stochastic fluctuations riding on a steady current. Therefore, mathematical models of TBL wall pressure take the form of a statistical space–time correlation function, and its corresponding Fourier transform or wavevector–frequency spectrum. Bull

* Corresponding author.
E-mail address: Anna.Caiazzo@kuleuven.be (A. Caiazzo).

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[1,2], Graham [3], Borisyuk and Grinchenko [4] and Hwang [5] made reviews of the semi-empirical models based on fits to experimental data under ideal conditions. From these studies, the Corcos model [6,7] has received large attention due to its advantageous mathematical features and its accurate description of the wavenumber–frequency spectrum of the wall pressure in the convective domain. Successively, other models have been developed in order to describe the low wavenumber range more accurately, still preserving a behaviour similar to the Corcos model in the vicinity of the convective peak. The most used are described by Chase [8], Williams [9] and Smol'yakov–Tkachenko [10]. From measurements reported by Farabee and Geib [11] and by Martin and Leehey [12], the Corcos wavenumber–frequency spectrum shows higher levels in the low wavenumber region than the experimental ones. On the contrary, models such as Chase or Smol'yakov–Tkachenko are more accurate in that range. However, they lack simplicity in the mathematical descriptions, making the Corcos model very versatile and used for certain applications, besides being the reference for other models.

The research presented in this paper starts with the observation that the Corcos model involves two Lorentzian functions to describe the pressure loading. In fact, when it comes to the integration over an infinite domain, the Lorentzian function allows mathematical simplifications that keep the Corcos model fully analytical in certain cases. For example, such advantageous mathematical features of the Lorentzian have been exploited in one dimension also by D'Amico [13] to perform Lorentzian-weighted frequency averaging for vibrating systems harmonically loaded.

The second key point, of the model proposed in this paper, is the use of a Butterworth filter instead of the Lorentzian. The Butterworth filter represents a generalization of the Lorentzian function and is characterized by its order. When the order is one, the Butterworth filter matches the Lorentzian. As shown by D'Amico, integrating the Butterworth filter over an infinite frequency range leads to a procedure that is analogous to the one used for the Lorentzian function [14,15]. Such a concept has been used to significantly speed up the computation time to evaluate band integrals for harmonically loaded vibro-acoustics systems [16,17].

In this paper, the novel model presented makes use of two Butterworth filters. On the one hand, this allows preserving the analyticity of the integration. On the other hand, by changing the order of the Butterworth filters it is possible to modify the rate of decay of the pressure distribution and improve the classic Corcos model in the low wavenumber domain. Therefore, the authors generalize the classic Corcos model focusing on the mathematical development of the novel model and on the comparison with the Corcos one keeping the same spanwise and streamwise coefficients.

The remainder of this paper is structured as follows. Section 2 gives a brief overview of the features of the wall pressure fluctuations of a TBL with focus on the early model proposed by Corcos in Section 3. In Section 4, the normalized wavenumber–frequency spectra for the models of Chase and Smol'yakov–Tkachenko are reviewed in order to underline their advantages and disadvantages. The Generalized Corcos model is proposed in Section 5 with its analytical development and its preliminary assessment is presented in Section 6. Section 7 illustrates an application case of a simply supported rectangular elastic plate driven by a TBL. The conclusions of the work are summarized in Section 8.

2. Features of the wall pressure fluctuations

The wall-pressure field associated with a TBL exhibits a random-like behaviour. Therefore, semi-empirical models, based on fits to experimental data under ideal conditions, are usually used to represent statistically the turbulent fluctuations. Those models are typically based on a spatial distribution, (*i.e.*, normalized cross-spectrum), a frequency distribution, (*i.e.*, auto-spectrum), and an absolute level, (*i.e.*, mean square wall pressure). For the representative case of a fluctuating wall-pressure field developed on a flat rigid surface in a low Mach number flow with zero mean pressure gradient, the TBL increases slowly in thickness in the flow direction. Under such conditions the generated pressure field can be regarded as homogeneous in space and stationary in time.

For a flow in the x direction over the (x,z) plane, the space–time cross correlation function of the pressure at two arbitrary space–time points (x,z) at time t, and $(x+\xi_x, z+\xi_z)$ at $t+\tau$ is given by

$$R_{pp}(\xi_x,\xi_z,\tau)=\langle p(x,z,t)p(x+\xi_x,z+\xi_z,t+\tau)\rangle, \tag{1}$$

where $\boldsymbol{\xi}\equiv(\xi_x,0,\xi_z)$ is the spatial separation vector and the brackets $\langle\rangle$ denote an ensemble average. Hence, the spatial cross-spectral density function (CSD) of the wall-pressure fluctuations is written as

$$\Psi_{pp}(\boldsymbol{\xi};\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}R_{pp}(\boldsymbol{\xi};\tau)\mathrm{e}^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau, \tag{2}$$

where ω is the radian frequency.[1] The CSD function in the wavenumber–frequency domain is defined as the spatial 2D Fourier transform of $\Psi_{pp}(\boldsymbol{\xi};\omega)$ upon spatial separation,

$$\Psi_{pp}(\mathbf{k};\omega)=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Psi_{pp}(\xi;\omega)\mathrm{e}^{-\mathrm{i}(\xi\mathbf{k})}\,\mathrm{d}\boldsymbol{\xi}, \tag{3}$$

where $\mathbf{k}\equiv(k_x,0,k_z)$ is the two-dimensional wavevector.

[1] Note that here the Fourier transform $\tilde{f}(\omega)$ of a function $f(t)$ is defined as $\tilde{f}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}f(t)\mathrm{e}^{-\mathrm{i}\omega t}\,\mathrm{d}t$ whereas other conventions can be used.