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Overview of coupling loss factors for damped and undamped simple oscillators

Dante Tufano [1], Zahra Sotoudeh [*,1]

Department of Mechanical, Aerospace, and Nuclear Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, United States

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ABSTRACT

This paper examines the coupling loss factor, which is an important parameter in the field of statistical energy analysis. A comparison is made between damped and undamped systems of two identical coupled oscillators using modal analysis. The oscillator energies and the power flow between oscillators are used to examine the transient coupling loss factors for the damped and undamped systems. Comparisons are drawn between the transient coupling loss factors and the steady-state coupling loss factor of classical SEA. The undamped transient coupling loss factor is determined numerically, since an analytical solution does not exist. It is shown that multiple formulations can be used to determine the transient coupling loss factor, and that certain formulations are preferable for moderately to strongly coupled systems.

1. Introduction

In this paper, the coupling loss factor (CLF) will be examined for damped and undamped systems of two-coupled oscillators. In statistical energy analysis (SEA), the CLF between two subsystems is defined as the ratio between the difference between time-averaged modal energies of the subsystems and the rate of energy flow between them. In the next section, the general concepts of SEA and the CLF will be outlined. The CLF is an important parameter in the field of statistical energy analysis [1–4]. Examining the CLF for undamped oscillators provides insight into whether SEA is a viable theoretical model for conservative systems. Currently, SEA provides valuable information about the rate of energy flow between nonconservative systems undergoing high-frequency vibration. The accuracy of the SEA model has not been examined in detail for conservative systems. Classical SEA is based on the conservation of energy, or the application of the first law of thermodynamics to structures. Since the equations of motion of coupled oscillators can be solved analytically, such systems are useful for validating SEA results, and have been studied extensively [3,5–8].

The goal of this paper is to examine and compare the CLFs for undamped and damped systems of coupled oscillators. For systems at steady state, the CLF is referred to as the *steady-state* CLF. Likewise, the *transient* CLF describes a system that is not at steady state. A detailed effort has not yet been made to examine the CLF for undamped oscillators. As such, there remains a need to evaluate the viability of applying SEA techniques to undamped systems. Such an analysis will provide insight into

* Correspondence to: Assistant professor, Department of Mechanical, Aerospace, and Nuclear Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, United States.

E-mail addresses: dantetufano@gmail.com (D. Tufano), zahra.vpi@gmail.com (Z. Sotoudeh).

[1] Tel.: +1 518 276 8145

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Nomenclature

E_i	energy of subsystem i
E_i^{diss}	dissipated power flow from subsystem i
E_i^{inj}	injected power flow into subsystem i
E_0	initial system energy
E_{tot}	total system energy
$\dot{E}_{i\to j}$	power flow from subsystem i to subsystem j
$\dot{E}_{ij}$	net power flow between subsystems i and j
$\frac{E_i}{N_{ii}}$	modal energy of subsystem i
$\hat{E}$	kinetic energy envelope of subsystem i
F_i	excitation force on subsystem i
N_i	number of modes for subsystem i
P_{ij}	time-average power flow between subsystems i and j
P_i	time-average of $\dot{E}_i$
T_i	kinetic energy of oscillator i
U_i	potential energy of oscillator i
$\hat{\dot{x}}_i$	envelope velocity of subsystem i
c	viscous damping constant
k	spring stiffness
m	oscillator mass
t	time
x_i	position degree of freedom of oscillator i
α	$\beta/2$
β	c/m
ΔE	difference between oscillator energies
$\Delta\omega$	SEA frequency band
ϵ	coupling strength
η_i	damping loss factor of subsystem i
η_{ij}	coupling loss factor (CLF) from subsystem i to j
$\eta_{ij}^{(s)}$	steady-state coupling loss factor between subsystems i and j
χ	$k\epsilon/m$
$\hat{\omega}$	beat frequency
ω	center of the frequency band $\Delta\omega$ blocked natural frequency for coupled oscillators
ω_i	ith natural frequency
$(*)^{(d)}$	damped system parameter
$(*)^{(u)}$	undamped system parameter
$\ddot{(*)}$	second derivative of "$(*)$" with respect to time (t)
$\dot{(*)}, \frac{d(*)}{dt}$	first derivative of "$(*)$" with respect to time (t)
$\langle(*)\rangle_t$	time-average of "$(*)$"
$:=$	equal by definition
$\approx$	approximately equal

the behavior of undamped coupled oscillators in relation to the more well-known results for damped oscillators. This paper considers deterministic systems of undamped and damped coupled oscillators that are each initially excited by a velocity impulse.

The governing equations of motion are solved for damped and undamped systems of coupled oscillators. Parameters are first determined for the undamped case and are then extended to the damped case. Expressions are obtained for the undamped and damped oscillator energies. Subsequently, "power flow" expressions are obtained for the undamped and damped systems. In the context of SEA, the term "power flow" refers to the time-rate of energy exchange between systems, or the time-derivative of the energy flow between systems. As such, the terminology does not refer to a flow of power, but to the rate of change of energy flow. Since the term "power flow" is commonly used in SEA, it is the notation that has been adopted in this paper [2,4–6]. Finally, multiple formulations for the transient CLF are determined using the time-averaged oscillator energy difference and power flow.

2. Review of the assumptions of statistical energy analysis

In SEA, physical structures are organized into subsystems. These subsystems are approximated as groups of oscillators. A SEA subsystem with a sufficient number of modes accurately approximates the spatial and time-averaged behavior of the actual system [21]. Only modes within a frequency band of interest, $\Delta\omega$, centered at ω, are considered. Each subsystem, i, has a corresponding energy, E_i, and number of modes, N_i. Typically, each subsystem exchanges energy with neighboring subsystems as well as the environment. The energy exchange rate between SEA subsystems is commonly referred to as the *power flow* in SEA publications [5].

For a SEA system composed of n subsystems, the power flow balance of the ith subsystem is [9]

$$\dot{E}_i = \dot{E}_i^{\text{inj}} - \dot{E}_i^{\text{diss}} + \sum_{i=1, i\neq j}^{n} \dot{E}_{i\to j}, \tag{1}$$

where $\dot{E}_i = \frac{dE_i}{dt}$ is the time-rate of energy change (power flow) of subsystem i, $\dot{E}_i^{\text{inj}}$ is the injected power, and $\dot{E}_i^{\text{diss}}$ is the dissipated power flow from subsystem i. $\dot{E}_{i\to j}$ is the power flow from subsystem i to subsystem j. Thus, the rightmost term of Eq. (1) represents the net power flow between oscillator i and all other oscillators. For steady-state SEA systems, $\dot{E}_i = 0$. The power balance in Eq. (1) is used in transient SEA (TSEA) [10–12], and will be used in this paper to determine the CLF for systems of coupled oscillators. The power flow due to dissipation of subsystem i, $\dot{E}_i^{\text{diss}}$, and the power flow to a subsystem j, $\dot{E}_{i\to j}$, can be expressed analogously using the damping loss factor (DLF) and CLF, respectively. In SEA, the power flow out of a system is taken to be proportional to the energy of the system. Accordingly, the DLF and CLF are constants of proportionality