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Uncovering inner detached resonance curves in coupled oscillators with nonlinearity

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ABSTRACT

Detached resonance curves have been predicted in multi-degree-of-freedom nonlinear oscillators, when subject to harmonic excitation. They appear as isolated loops of solutions in the main continuous frequency response curve and their detection may thus be hidden by numerical or experimental analysis. In this paper, an analytical approach is adopted to predict their appearance. Expressions for the amplitude-frequency equations and bifurcation curves are derived for a two degree-of-freedom system with cubic stiffness nonlinearity, and the effect of the system parameters is investigated. The interest is specifically towards the occurrence of closed detached curves appearing inside the main continuous frequency response curve, which may lead to a dramatic reduction of the amplitude of the system response. Both cases of hardening and softening stiffness characteristics are considered. The analytical findings are validated by numerical analysis.

1. Introduction

Detached resonance curves (DRCs) appear as isolated loops of solutions in the frequency response curves (FRCs) of oscillating systems with nonlinearity. Their detection may be undisclosed either when applying classical numerical techniques or when performing sine-sweep experimental tests. An analytical approach is then convenient because it would show the effects of the main system parameters on the DRCs appearance. Furthermore, it would be helpful to predict DRCs in advance, prior to conduct any numerical or experimental analysis.

DRCs manifest as a result of multivaluedness in the FRC [1]. This means that, in case of harmonic excitation and provided the system response is predominantly harmonic at the excitation frequency, multiple solutions may appear in the steady-state amplitude responses at a single frequency. Depending on the values of the system parameters, multivaluedness can lead to closed DRCs, which can either lay inside or outside the main continuous FRC.

Analyzing the performance of a vibration absorber with a nonlinear damping characteristics attached to a linear host structure, Starosvetsky and Gendelman [2] reported theoretical prediction and numerical confirmation of outer DRCs. The same authors predicted similar features when analyzing a three degree-of freedom (DOF) nonlinear oscillating system [3]. The values of the system parameters greatly affect the appearance of these features, so that, for instance, in the two DOF experimental setup tested in [4] DRCs did not manifest. Alexander and Schilder [5] found out a family of outer DRCs for vanishing linear spring stiffness term when analyzing the performance of a nonlinear tuned mass damper with linear plus cubic stiffness nonlinearity. Detroux et al. [6] identified DRCs numerically in the forced response of a satellite structure.

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Kuether et al. [7] studied the connection between nonlinear normal modes and isolated resonance curves appearing outside the main frequency response curve of a forced nonlinear system. A family of isolated sub-harmonic branches in the nonlinear frequency response of an oscillator with clearance was studied in [8], and outer detachments were also observed in case of impact phenomena [9]. By studying the response of a harmonically excited system consisting of coupled linear and nonlinear oscillators with hardening characteristics, Gatti and Brennan [10] predicted the appearance of either inner or outer DRCs and investigated the effect of parameters in the appearance of such features. They studied the system under the assumption of very small ratio between the mass of the attachment and that of the main structure. An experimental test was also conducted by the use of a very heavy electro-magnetic shaker and a relative light mass for the attachment [11], but the physical system parameters were not suitable to uncover a DRC experimentally. In that case, it was evident that inner DRC manifests for low values of the linear stiffness and damping in the attachment. Despite the theoretical formulation used in [10] and [11] captures physically the phenomena of DRCs, its practical application to real engineering design problems is limited by the former assumption about the mass ratio.

To the best of the author's knowledge it seems that there is no comprehensive work which clearly shows the effects of the main parameters of a nonlinear oscillating system on the appearance of such features, and the conditions and limitations for those features to emerge in the main continuous FRC. Thus, with the aim to provide a series of analytical usable expressions related to such interesting phenomenon, this paper presents a detailed theoretical analysis on the response of a harmonically excited two DOF oscillating system with coupling nonlinear stiffness. By removing previous limitations, as the assumption of small mass ratio [11] or the assumption of a light primary suspension [12], a novel and complete closed-form solution for the amplitude-frequency response equation and detachments is derived using the harmonic balance method. Both the cases of nonlinear hardening and softening spring are considered and it is shown how this characteristics affects the occurrence of a DRC and the corresponding ranges for the values of the system parameters.

Uncovering the main mechanism underneath the appearance of DRC will help engineers to better design nonlinear system (or system expected to operate in nonlinear conditions) and thus avoiding (or exploiting) a detachment to arise in the frequency response with a consequent dramatic amplitude shift.

The theoretical approach adopted in this work can be extended to different configurations of nonlinear coupled oscillators, so as to investigate their specific dynamic behavior when subject to harmonic excitation.

2. System description and equations of motion

The system of interest in this work is shown in Fig. 1. A primary oscillating mass m_s is connected to ground through a linear spring k_s and damper c_s and it is excited by a force f. A secondary mass m is attached to the first mass through a linear damper c_1 and a nonlinear spring, which consists of a linear, k_1, plus cubic, k_3, stiffness term.

The equations of motion of the system may be written in terms of the displacement of the primary mass, x_s, and the relative displacement between the two masses, $z=x_s-x$, where x is the displacement of the secondary mass. They are given below as

$$m_s\ddot{x}_s+c_s\dot{x}_s+k_sx_s+c_1\dot{z}+k_1z+k_3z^3=f(t)$$
$$m\ddot{x}_s-m\ddot{z}-c_1\dot{z}-k_1z-k_3z^3=0 \tag{1a,b}$$

where the overdots denote differentiation respect to time t.

If the primary mass is driven by a harmonic force with constant amplitude F at each frequency ω , i.e. $f(t)=F\cos(\omega t)$, Eq. (1a,b) can be conveniently written in non-dimensional form as

$$y_s''(1+\mu)+2\zeta_s y_s'+y_s=\cos(\Omega\tau)+\mu w''$$
$$w''+2\zeta w'+\Omega_0^2w+\gamma w^3=y_s'' \tag{2a,b}$$

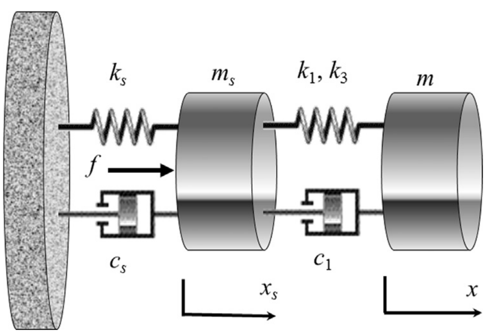

Fig. 1. Model of the two DOF nonlinear system.