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Spatial resolution limits for the localization of noise sources using direct sound mapping

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ABSTRACT

One of the main challenges arising from noise and vibration problems is how to identify the areas of a device, machine or structure that produce significant acoustic excitation, i.e. the localization of main noise sources. The direct visualization of sound, in particular sound intensity, has extensively been used for many years to locate sound sources. However, it is not yet well defined when two sources should be regarded as resolved by means of direct sound mapping. This paper derives the limits of the direct representation of sound pressure, particle velocity and sound intensity by exploring the relationship between spatial resolution, noise level and geometry. The proposed expressions are validated via simulations and experiments. It is shown that particle velocity mapping yields better results for identifying closely spaced sound sources than sound pressure or sound intensity, especially in the acoustic near-field.

1. Introduction

Over the last decades there has been dramatic progress in the development of acoustic imaging techniques [1–3]. The transformation of sound into something visual is considered key to understanding a wide variety of problems. Many techniques and apparatus have been proposed over the years, most with a common goal: localize where sound originates. Although noise can be the result of a complex chain of events, finding the areas of a machine or structure that create a significant acoustic excitation is a good starting point for applying appropriate noise control measures.

A large number of methods have been developed for pressure microphone arrays [4]. Near-field acoustic holography [5], acoustic beamforming [6] and various inverse methods [7] offer different approaches to localize, and ultimately quantify, the sources of noise. However, pressure-based techniques often encounter difficulties adapting from controlled experiments to industrial applications [8]. In many cases the presence of various critical factors such as source dimensions, room reverberation or noise produced by surrounding machinery may increase estimation error, ultimately limiting the ability to resolve the assessed sound sources accurately [9,10].

The direct visualization of sound, in particular sound intensity, has been used extensively since the 1980s to locate, quantify and rank sound sources [4]. Ever since the introduction of sound intensity probes a series of standardized measurement methods have been available for performing the in situ characterization of complex sound sources. Despite its

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simplicity, the direct representation of sound intensity has proven very useful for a wide range of practical cases. Although there is an extensive amount of literature that covers the foundations of sound intensity for noise quantification [11,12], a detailed description of the spatial resolution limit for direct sound intensity mapping is still not yet defined.

In recent years, the introduction of new mapping techniques which enable the rapid visualization of spatial sound distributions [13–15] has been applied to multiple industrial problems [16–18], increasing the interest in exploring the foundations of the direct representation of sound. This paper evaluates several spatial resolution criteria and introduces a novel model to establish the resolution limits of direct sound pressure, acoustic particle velocity and sound intensity mapping.

2. Resolution criteria

Several criteria can be used to describe when two sources should be regarded as resolved. Since resolution is not unambiguously defined, various interpretations have been proposed during the last two centuries, most of them originally introduced in the field of optics [19–21]. The natural analogy between acoustics and optics is explored in this section to derive the most popular resolution criteria using a well-defined technique such as acoustic beamforming. The assumptions described in this section are hence independent of the underlying theory of direct sound mapping. However, the results obtained will be used to understand what can be considered as resolvable by applying an objective quantification of spatial resolution to the output signal.

The derivation that follows considers the limits of a transducer array sensing incoherent sound waves using acoustic beamforming [22,23]. Considering a continuous line array located along the x-axis of size D, when a wave front with a wavenumber vector $\mathbf{k}=[k_x,k_y,k_z]$ impinges upon the array it produces an output proportional to the spatial integral over the aperture [7]

$$W(\mathbf{k})=\int_{-D/2}^{D/2} b(\mathbf{r})\Phi(\mathbf{r},\mathbf{k})\,\mathrm{d}x, \tag{1}$$

where $b(\mathbf{r})$ represents a spatial weighting function and $\Phi(\mathbf{r},\mathbf{k})$ denotes the quantity being measured along the array at position $\mathbf{r}$. For the simple case of a plane wave travelling towards a uniform linear array of sound pressure sensors located at $\mathbf{r}=[x,0,0]$

$$\Phi(\mathbf{r},\mathbf{k})=e^{j(\mathbf{k}\cdot\mathbf{r})}=e^{jk_x x}. \tag{2}$$

Using a uniform weighting $b(\mathbf{r})=1/D$, and combining Eqs. (1) and (2) the array output produces an interference pattern that varies following a sinc function such as

$$W(\mathbf{k})=\frac{\sin\,(k_x D/2)}{k_x D/2}. \tag{3}$$

The array pattern can be steered towards a certain direction $\mathbf{k}_0$ by evaluating $W(\mathbf{k}-\mathbf{k}_0)$. As a result, the combination of signals captured within the aperture will yield the highest output when it is steered towards the sound direction of arrival.

The definition of the array output presented in Eq. (3) is assessed below using the main resolution principles commonly used: the Rayleigh and the Full Width at Half Maximum (FWHM) criteria. The main differences between these two methods are then studied using the Valley to Peak ratio (V/P ratio). The derivations that follow are focused on the particular case when two uncorrelated wavefronts travelling in two slightly different directions are sensed by a uniform linear aperture as two different sources.

2.1. Rayleigh limit

One classical definition of resolution is the Rayleigh criterion [24]. It is assumed that a pair of incoherent plane waves can be resolved if the shifted peak of the aperture smoothing function $W(\mathbf{k}-\mathbf{k}_1)$ falls on the first zero when pointing towards the other direction of propagation $W(\mathbf{k}-\mathbf{k}_2)$. Therefore, the Rayleigh resolution is equal to the smallest wavenumber that produces a zero in the array pattern $W(\mathbf{k})$, i.e.

$$\min(k_x)=k\,\sin\,(\theta_{\min})=\frac{2\pi}{D}. \tag{4}$$

The following relationship can be established by focusing the array on a source plane located at a distance d

$$\sin\,(\theta_{\min})=\frac{R_R}{\sqrt{R_R^2+d^2}}, \tag{5}$$

where R_R denotes the minimum resolvable distance between sources according to the Rayleigh criterion. Given that