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Harmonic response of multilayered one-dimensional quasicrystal plates subjected to patch loading

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ABSTRACT

Dynamic analyses of a multilayered one-dimensional quasicrystal plate subjected to a patch harmonic loading with simply supported lateral boundary conditions are presented. The pseudo-Stroh formulation and propagator matrix method are used to obtain the exact three-dimensional response of the plate. In order to avoid resonance, the frequency of the patch loading is chosen away from the natural frequencies by introducing a small imaginary part. The patch loading is expressed in the form of a double Fourier series expansion. Comprehensive numerical results are shown for a sandwich plate with two different stacking sequences. The results reveal the influence of layering, loading area, phonon–phason coupling coefficient and input frequency. This work is the first step towards understanding quasicrystals under intricate loading conditions such as indentation and impact, and the exact closed-form solution can serve as a reference in convergence studies of other numerical methods and for verification of existing or future plate theories.

1. Introduction

One group of complex metal alloys, called quasicrystals (QCs), was first discovered by Shechtman in the early 1980s from the diffraction image of rapidly cooled Al-Mn alloys [1]. This discovery was revolutionary for it showed that QCs exhibit symmetries that are contradictory to the classical crystalline law of symmetry. Classical crystals are composed of particles assembled in a unique periodic arrangement in space and must hold two-, three-, four- or six-fold rotational symmetry. QCs, on the other hand, can be both ordered and nonperiodic which form patterns that lack translational symmetry [2]. QCs in the real three-dimensional physical space may be seen as a projection of a periodic lattice in the higher dimensional mathematical space. The projection of the periodic lattice in four-, five-, and six-dimensional space to the physical space generates one-, two- and three-dimensional QCs, respectively. One-dimensional (1D) QCs, considered in this work, refer to a three-dimensional structure with atomic arrangement which is quasiperiodic in one direction and periodic in the plane perpendicular to that direction.

Attributing to its nonperiodic atomic structure, QCs possess properties such as corrosion resistivity, low thermal conductivity, low coefficients of friction, low porosity, high hardness, and high wear resistance. These properties have enabled

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QCs to be applied as thin films and coatings [3]. Due to their advantageous properties, QCs have gained considerable interest in a wide range of study fields. However, phonon–phason coupling, anisotropy, and nonsymmetry intrinsic in quasicrystalline materials present many obstacles to researchers such that most problems are limited to cracks in a half space [2,4,5]. Since plates are of vital importance in structural design [6], plates made of QCs have been studied considerably. These include works pertaining to the three-point bending of QC thin plates under static and transient dynamic loads [7], static [8] and free vibration response [9] of multilayered QC plates, and thick QC plate analysis using refined plate theory [10].

In this work, the exact closed-form solution of a multilayered plate made of 1D QCs subjected to harmonic patch surface loading under laterally simply supported conditions is presented. Patch surface loading is selected in this dynamic analysis for it is a common loading case that closely reflects reality and an arbitrary distributed load can be approximated by a number of patch loads. The pseudo-Stroh formalism and the propagator matrix method [11] are utilized in obtaining the exact solution. Part of the distinction in the application of the pseudo-Stroh formalism and the propagator matrix method in this work pertains to the explicit double Fourier series expansion of the exact solution, instead of selecting only one Fourier component as in previous works. Since a sandwich plate is the most common structure in the research of multilayered plates, numerical illustrations of sandwich plates subjected to uniformly distributed harmonic patch surface loading are presented. The effects of stacking sequence, input frequency, patch loading area, and phonon–phason coupling on the harmonic response of sandwich plates are investigated. To the best of the authors' knowledge, analyses of multilayered three-dimensional QC plates under harmonic patch loading conditions have not been previously reported in the literature. Considering that QCs are used in structures which may be subjected to indentation and impact, this work is essential towards the understanding of QCs under such complex loading. As the solution presented in this work is based on the exact three-dimensional elasticity, there is no assumption on the aspect ratio commonly used in plate theories. In this sense, the numerical results presented in this work can serve as a benchmark for various numerical methods in layered QC analysis, and are further important for checking the accuracy of existing or any future plate theories.

2. Quasicrystalline elastic theory

In this section, the fundamentals of linear elastic theory for QCs are described. Two different elementary excitations are associated with atomic motion in QCs: phonons and phasons. Phonon displacement u_i is related to translation of atoms whereas phason displacement w_i is related to atomic rearrangements along the quasiperiodic direction. Both displacement fields are needed in the analysis of QCs and are actually coupled with each other [2,3,12].

From the linear elastic theory of QCs [12], the strain-displacement relations are

$$\begin{aligned} \varepsilon_{ij} &= \frac{1}{2}(\partial_j u_i + \partial_i u_j) \\ w_{ij} &= \partial_j w_i \end{aligned} \tag{1}$$

where ε_{ij} is the phonon strain tensor and w_{ij} is the phason strain tensor. The generalized constitutive relations of quasicrystalline materials are [12],

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} + R_{ijkl}w_{kl} \\ H_{ij} &= R_{klij}\varepsilon_{kl} + K_{ijkl}w_{kl} \end{aligned} \tag{2}$$

where σ_{ij} is the phonon stress tensor, C_{ijkl} phonon elastic coefficients, H_{ij} phason stress tensor, K_{ijkl} phason elastic coefficients, R_{ijkl} phonon-phason coupling coefficients, and repeated indices indicate the summation over 1, 2, and 3 (or over x, y, and z). It should be noted that although for 1D QCs the phonon stress tensor is symmetric, the phason stress tensor is not. Similarly, the phonon strain tensor is symmetric whereas the phason strain tensor is not [2].

Considering a 1D QC with x- and y-axes as the periodic directions, it follows that the z-axis is the quasiperiodic direction. Since the z-axis is the quasiperiodic direction, there is no phason displacement in the plane normal to this direction. Accordingly, $w_x = w_y = 0$. For a hexagonal system and Laue class 10 with point groups $62_h2_h, 6mm, \overline{6}m2_h$, and $6/m_hmm$ the linear constitutive relations in Eq. (2) are expanded as [2]

$$\begin{aligned} \sigma_{xx} &= C_{11}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + C_{13}\varepsilon_{zz} + R_1 w_{zz} \\ \sigma_{yy} &= C_{12}\varepsilon_{xx} + C_{11}\varepsilon_{yy} + C_{13}\varepsilon_{zz} + R_1 w_{zz} \\ \sigma_{zz} &= C_{13}\varepsilon_{xx} + C_{13}\varepsilon_{yy} + C_{33}\varepsilon_{zz} + R_2 w_{zz} \\ \sigma_{yz} &= 2C_{44}\varepsilon_{yz} + R_3 w_{zy} \\ \sigma_{xz} &= 2C_{44}\varepsilon_{xz} + R_3 w_{zx} \\ \sigma_{xy} &= 2C_{66}\varepsilon_{xy} \\ H_{zz} &= R_1(\varepsilon_{xx} + \varepsilon_{yy}) + R_2\varepsilon_{zz} + K_1 w_{zz} \\ H_{zx} &= 2R_3\varepsilon_{xz} + K_2 w_{zx} \\ H_{zy} &= 2R_3\varepsilon_{yz} + K_2 w_{zy} \end{aligned} \tag{3}$$

with $C_{66} = (C_{11} - C_{12})/2$. We point out that the forth-order tensor C_{ijkl} is condensed to a second-order tensor C_{ij} via Voigt notation with the standard mapping between indices as (11)→(1), (22)→(2), (33)→(3), (23)→(4), (13)→(5) and (12)→(6).