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Global axial–torsional dynamics during rotary drilling

Sunit K. Gupta*, Pankaj Wahi

Mechanical Engineering, Indian Institute of Technology Kanpur, India

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ABSTRACT

We have studied the global dynamics of the bottom hole assembly (BHA) during rotary drilling with a lumped parameter axial–torsional model for the drill-string and a linear cutting force model. Our approach accounts for bit-bounce and stick-slip along with the regenerative effect and is independent of the drill-string and the bit–rock interaction model. Regenerative axial dynamics due to variable depth of cut is incorporated through a functional description of the cut surface profile instead of a delay differential equation with a state-dependent delay. The evolution of the cut surface is governed by a nonlinear partial differential equation (PDE) which is coupled with the ordinary differential equations (ODEs) governing the longitudinal and angular dynamics of the BHA. The boundary condition for the PDE captures multiple regeneration in the event of bit-bounce. Interruption in the torsional dynamics is included by considering separate evolution equations for the various states during the stick period. Finite-dimensional approximation for our coupled PDE-ODE model has been obtained and validated by comparing our results against existing results. Bifurcation analysis of our system reveals a supercritical Hopf bifurcation leading to periodic vibrations without bit-bounce and stick-slip which is followed by solutions involving bit-bounce or stick-slip depending on the operating parameters. Further inroads into the unstable regime leads to a variety of complex behavior including co-existence of periodic and chaotic solutions involving both bit-bounce and stick-slip.

1. Introduction

Exploration of fossil fuels like oil and natural gas relies heavily on deep drilling systems. Deep drilling systems can be classified as rotary drilling, percussive drilling or a combination of both [1–4]. However, rotary drilling has emerged as the most commonly used and cost-effective technique for drilling oil wells. Schematic of a typical drilling set-up for rotary drilling is shown in Fig. 1. It consists of a rotary table at the surface with a power transmission which transmits the rotary power required for drilling to the bottom hole assembly (BHA) which carries the drill–bit with the cutter through a series of drill pipes called the drill-string [5]. One of the phenomena plaguing drilling operations is self-excited vibrations which leads to drill-string failure such as fatigue [6,7]. Therefore it is necessary to understand the dynamics and behavior of the drill-string during vibrations to avoid or reduce failures. Drill-string vibrations can be divided in three broad categories – axial, torsional and lateral which further lead to bit-bounce, stick-slip and whirl phenomena, respectively. There is an intricate coupling between these vibration modes and several studies have been devoted to better understand the full

* Corresponding author. Tel.: +91 512 2596092.
E-mail addresses: sunitgpt@iitk.ac.in (S.K. Gupta), wahi@iitk.ac.in (P. Wahi).

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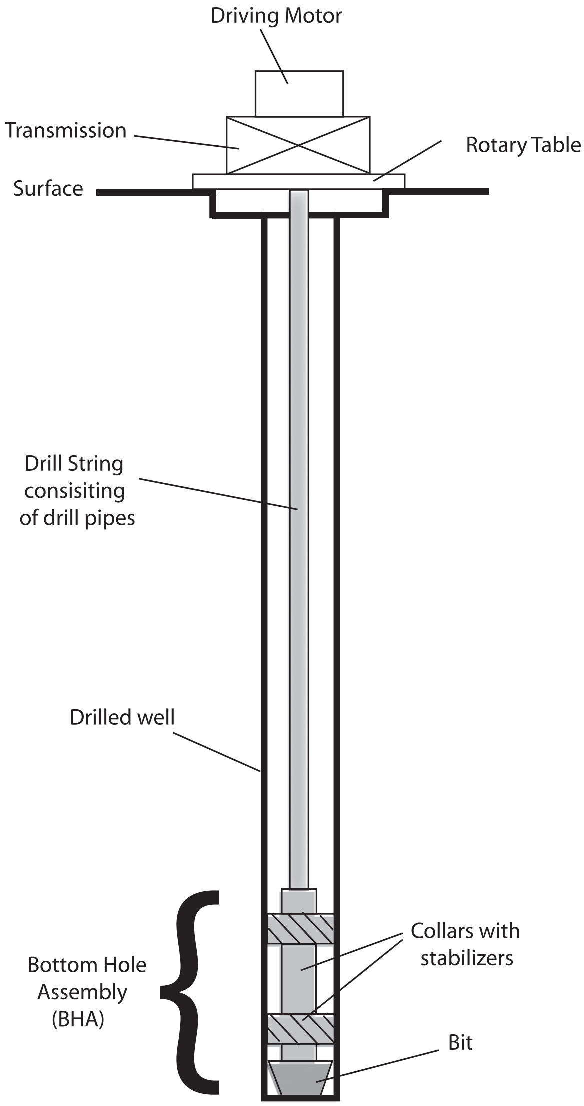

Fig. 1. Schematic of a rotary drilling set-up.

dynamics of the rotary drilling systems [8–11]. In this paper, we consider the coupled axial and torsional vibrations of a rotary drilling system and explore parameter regimes involving excessive vibrations leading to bit-bounce (loss of contact between the drill–bit and the surface) and/or stick-slip (intermittent seizure of angular motion of the drill–bit).

One of the first approaches towards modeling the rotary drilling process was presented by Bailey and Finnie [12] who obtained the natural frequencies of axial and torsional vibrations of the drill-string from independent one-dimensional wave equation. This was followed by their experimental studies [13] in which they found the measured force, torque, and natural frequencies of the system to have some correspondence with the theoretical study [12] for an appropriate choice of the boundary conditions. Paslay and Bogy [14] specified a sinusoidal motion of the drill–bit as a representative motion during longitudinal vibrations and obtained the required variable bit forces using the mobility method. Dareing and Livesay [15] later found that a distributed damping along the length of the drill-string is critical in obtaining a good match between the theoretical and experimental results for the displacements and forces during axial and torsional vibrations. Lateral vibrations of the drill string and the associated axial buckling load were considered for the first time by Huang and Dareing [16] in 1968 who modeled the drill string as a simply supported beam. Dunayevsky et al. [17] studied lateral vibrations of the drill strings with a fluctuating axial force and found the instabilities corresponding to parametric excitation. Dawson et al. [18] in 1987 were the first to discuss stick-slip phenomenon during torsional drill-string vibrations wherein they obtained the minimum value of rotary speed for sticking to occur. Similarly Spanos et al. [19] were the first to incorporate bit-bounce during excessive longitudinal vibrations of a drill-string with the roller cone bit. They studied the relationship between weight on bit (WOB) and the amplitude of the bit-motion leading to bit lift-off and observed that the rotary speeds corresponding to axial resonant frequencies of the system are critical.