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Properties of stability, bifurcation, and chaos of the tangential motion disk brake

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ABSTRACT

This study proposes a new dynamic model of a brake system that combines pad tangential motion and disk torsional motion to reduce the vibration and noise of the brake system. The stability analysis of this system with a smoothed Stribeck friction model verified its instability, which is caused by Hopf bifurcation. Moreover, numerical simulation showed several phenomena of the system vibration changing with angular velocity: (1) the system vibration maintains in the stable limit cycle after Hopf bifurcation within a relatively wide range of low angular velocity and (2) period-doubling bifurcation and chaos will occur only by decreasing the angular velocity. This study further discusses the effects of friction parameter on stick–slip vibration within a common range of brake pressure under "low" and "lower" angular velocities; the result shows that a decrease of both $\mu_s - \mu_k$ and decay factor can effectively reduce the range of chaotic vibration region.

1. Introduction

Noise of a brake system caused by vibration is one of the challenges in the automotive industry. Many manufacturers of brake pad materials spend more money on resolving it [1,2]. Numerous scholars have conducted many modeling works to explore the mechanism of vibrations, and their models could be classified into two categories: continuous and discrete. The continuous model treats the brake components as a flexible system, either treating the rotor as a flexible annular disk [3,4] or the pad as a flexible Euler–Bernoulli beam [5,6], whereas the discrete model treats them as lumped masses. However, the continuous model was mainly used for the analysis of transverse vibration, and the tangential vibration focused on this study is an in-plane vibration. Thus, the discrete model proves to be more appropriate for this study.

Recent achievements in friction-induced vibration on the discrete model are the facts that (1) the stick–slip mechanism induced by dry friction is one of the most significant reasons for the occurrence of brake vibrations at low speed, (2) the necessary condition for stick–slip vibration would be the negative correlation between friction and velocity [7,8], particularly the existence of a boundary value of relative velocity: the stick–slip motion will occur only when the relative velocity is less than the boundary value of relative velocity [9,10].

Many scholars have studied the single degree of freedom (SDOF) of slider–sliding belt model; however, it only considers the pad vibration by neglecting the disk effect. Shin [11,12] simplified the physical model into a one DOF system model coupled with a friction pair for disk and pad, and then studied the effect of system damping on stick–slip motions, which

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reveals that the damping effect of the disk and pad is also crucial to the stability of the brake system. Yang [13] discussed the characteristics of period-doubling bifurcation and chaos of the brake system for various disk speeds by applying a new dry friction model. Nevertheless, Paliwal [14] believed that the interfacial coupling stiffness between the disk and pad will change during braking, and these effects on stick–slip motions can never be neglected. Thus, he improved the model by increasing the coupling stiffness between the disk and pad on Shin's model, and then studied the impact of the coupling stiffness on the stability and stick–slip motions of the brake system. However, each of the aforementioned dynamic models only considers the tangential vibration.

In fact, because the disk motion essentially belongs to rotary motion, it is more appropriate to denote the vibration as a torsional motion rather than a tangential motion for modeling the disk motion. Crowther [15,16] proposed a four-DOF torsional model by combining the driveline (the power plant, disk, and tire) and the brake torsional subsystems coupled with a friction pair. He discussed the stick–slip motions of the coupled system under low constant drive torque according to Coulomb's law, and found three types of stick–slip motions with different brake pressures. Zhang [17] simplified this four-DOF torsional model into a two-DOF model. He analyzed the periodic stick–slip motion of the system under different driving speeds, and found that a higher driving speed will result in a longer slip phase and higher stick–slip frequency. Li [18] developed an SDOF torsional model of a wedge brake with harmonic excitation from driveline, and investigated the effect of actuation force and wedge angle on the system vibration. He also compared the dynamic responses of the wedge and conventional brakes. However, the aforementioned dynamic models only consider the torsional motion. In fact, because either of the two models (i.e., tangential and torsional models) benefits either the pad motion or disk motion, both of which are significant parts of the brake system, a combination of both models will better reflect the real vibration of a brake system.

In this study, we propose a new dynamic model that comprehensively considers the pad tangential vibration and disk torsional vibration and simplify the vibration into a two-DOF system coupled with a friction pair (Fig. 1). Then, a stability analysis is conducted by applying a smoothed Stribeck friction model [19,20], and found that the property of the Hopf bifurcation of the system alters with the angular velocity, which is verified by numerical calculation. Finally, we discuss the effect of friction parameters on the stick–slip vibration under different angular velocities within an accepted range of brake pressure.

2. Dynamic model of the brake system

2.1. Mechanical model and equations of motion

The existing dynamic brake models consider purely the tangential motion of the brake pad by either simplifying the disk as a belt or reducing both the pad and disk as either tangential or torsional motion. In fact, the motion of the disk essentially belongs to the rotation while the pad's motion corresponds to the translation. Therefore, we establish a brake system that combines the respective tangential and torsional motions of the pad and disk.

Fig. 1 shows the mechanical model of a brake system, where m_b represents mass of the pad, which exhibits tangential motion and J_r represents inertia of the disk, which exhibits torsional motion: these two motions are coupled with a friction pair. The parameters k_b, c_b and k_r, c_r represent the stiffness and damping of the pad and disk, respectively, x_b and θ_r denote the tangential displacement of the pad and the torsional angular displacement of the disk, respectively, r_b represents the distance between the pad and the center of the disk (hereafter referred to as friction radius), v_r is the relative velocity between the pad and disk, ω denotes the angular velocity input exerted on the disk, F_N is the brake pressure applied on the pad, F_b is the friction force acting on the pad, and T_b represents the friction torque on the disk.

On the basis of Newton's law, the mechanical model (Fig. 1) is represented by two differential equations of motion:

$$\begin{cases} J_r\ddot{\theta}_r + c_r\dot{\theta}_r + k_r\theta_r = T_b \\ m_b\ddot{x}_b + c_b\dot{x}_b + k_b x_b = F_b \end{cases}, \tag{1}$$

where the characteristics of F_b and T_b depend on the property of the friction model.

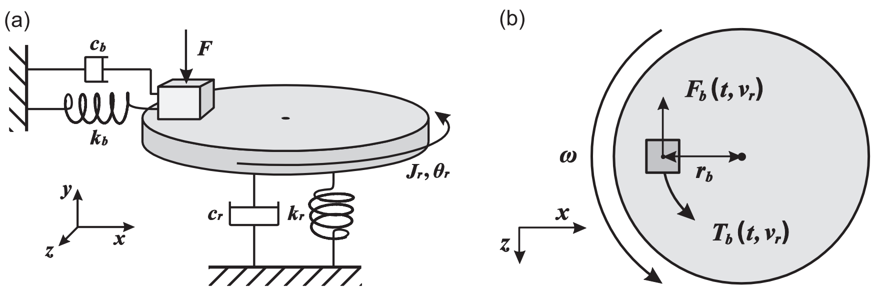

Fig. 1. Mechanical model of a brake system.