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Analytical expressions for chatter analysis in milling operations with one dominant mode

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ABSTRACT

In milling, an accurate prediction of chatter is still one of the most complex problems in the field. The presence of these self-excited vibrations can spoil the surface of the part and can also cause a large reduction in tool life. The stability diagrams provide a practical selection of the optimum cutting conditions determined either by time domain or frequency domain based methods. Applying these methods parametric or parameter traced representations of the linear stability limits can be achieved by solving the corresponding eigenvalue problems. In this work, new analytical formulae are proposed related to the parameter domains of both Hopf and period doubling type stability boundaries emerging in the regenerative mechanical model of time periodical milling processes. These formulae are useful to enrich and speed up the currently used numerical methods. Also, the destabilization mechanism of double period chatter is explained, creating an analogy with the chatter related to the Hopf bifurcation, considering one dominant mode and using concepts established by the Pioneers of chatter research.

1. Introduction

The first theories explaining machining chatter vibrations as a self-excited regenerative phenomenon were developed by Tlusty and Polacek [1] and Tobias and Fishwick [2]. Later on, Merrit [3] presented the problem as a feedback loop, clarifying the problem from an engineering point of view. The corresponding mathematical models are delay-differential equations (DDE) and the stability properties can be studied through these DDEs [4]. For the simplest case of turning, the governing equation is an autonomous DDE with constant parameters including a directional factor, which is used to project the cutting forces onto the mode direction and the vibration onto the chip thickness direction [1,2]. In this case, the presence of chatter is related to sub-critical Hopf bifurcation [4–6].

The discontinuous and non-autonomous nature of the milling process introduces additional difficulties to the stability analysis. In milling, one or more rotating cutting edges are subject to time dependent local forces, causing the resultant cutting force to vary in magnitude and direction time-periodically. In milling processes, the governing equation is a time periodic piecewise smooth delay differential equation (DDE) [4]. Moreover, unlike turning operations, multiple chatter frequencies arise as a combination of the main chatter frequency and the tooth passing frequency, according to Floquet theory [5]. Stability analysis is therefore much more complex than those representing continuous regenerative cutting processes.

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Minis and Yanushevsky [7] formulated a direct solution to the stability of the milling problem in frequency domain. They used Floquet's theorem and the Fourier series on a two-degree-of-freedom cutting model and defined a characteristic equation based on an infinite determinant dependent on chatter frequency and tooth passing frequency. Based on a zero order approximation or truncation of this numerical method, Altintas and Budak [8] developed a parametric analytical determination of stability limits, which allows a rapid calculation of the stability lobe diagram. The zero order approximation model (ZOA) offers accurate results with low oscillation milling force and it loses its accuracy as more harmonics are involved in the time periodic resultant cutting force. In this more interrupted case, the results determined by ZOA differ from those seen in experiments. This is due to the presence of additional double-period lobes related to flip bifurcation [9] and mode interaction related to Hopf bifurcation when the system has more than one dominant mode [10].

The stability of interrupted cutting has been determined by time domain based alternative methods. Davies et al. [9] used a discrete map model for highly interrupted milling processes, where the time in cut is considered infinitesimal and modelled as an impact. Mann et al. [6] obtained similar results using time domain finite elements. Insperger and Stépán developed the semi-discretisation [11,12] technique for DDE which can generally capture different kind of stability losses of stationary (forced) vibration appearing in the real milling process. These methods permit to obtain the stability of complex tool geometries for their optimisation [13,14]. The double-period chatter can be also predicted in frequency domain by multi-frequency solutions [15,16] solved through numerical iterations.

In general, the previously mentioned time domain based methods [6,11,12] are substantiated on numerical discretisation and are more accurate and time consuming than the zero order approximation. These time domain based methods are scanning different conditions of the stability chart, usually depth of cut and spindle speed, directly assessing stability point by point. Therefore, there is a need for shortening the calculation time of this time domain based methods while retaining the accuracy of the prediction. The use of analytical approximate expressions or formulas can help to define the boundaries of time domain based accurate methods analytically, thus increasing their efficiency.

Regarding chatter related to Hopf bifurcation, the pioneers [1–3] defined approximate formulas for the case of one dominant mode. In addition, the solution proposed by Altintas and Budak [8] permits a fast approximate result that can be used to define the boundaries for more accurate methods. Recently, Warminski et al. [17] also proposed approximate analytical solutions considering nonlinear friction terms.

However, the case of the double period chatter and flip bifurcations is not completely solved in the literature. Corpus and Endres [18] studied in detail the double period lobes in a one-degree of freedom system and proposed an analytical expression for the high speed double period lobe and its asymptotes. Zatarain obtained a formula to calculate the intersection point of the branches defining the flip lobes and detailed a graphical representation useful to obtain the starting frequency of flip chatter [19]. Nevertheless, there are not analytical formulas available for the definition of the double period chatter minimum.

The objective of this work is to improve the knowledge about double period chatter offering new simple and fast analytical mathematical expressions useful to analyse the stability and predict the relative importance of the flip bifurcation in a certain milling operation. A comparison with traditional Hopf related chatter is also carried out using the concepts defined by Pioneers in chatter theories, in order to clarify the connections between cutting process and dynamics for period doubling related chatter. The procedure defined in this work can be extrapolated to other vibratory problems ruled by DDE as a second order periodic robotic system with time delay under a PD control [20].

2. General multi-frequency formulation in modal coordinates for one dominant mode

A simple form of the multi-frequency solution is presented in this section for the milling operation in modal coordinates.

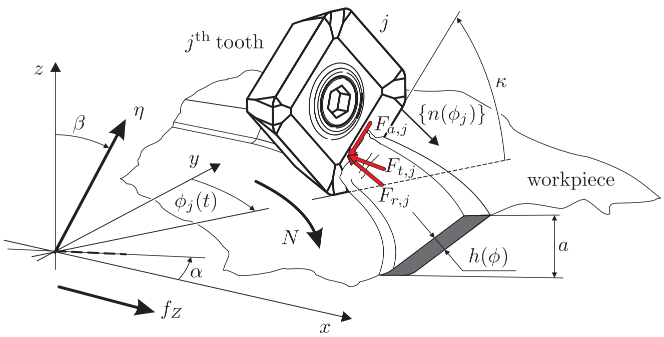

Fig. 1. Geometry of the face milling cutter with local (t, r, a) cutting forces. The direction of mode n is defined with spherical coordinate system (α, β, η) in the Cartesian coordinate system (x, y, z).