Journal of Sound and Vibration 371 (2016) 196–209

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Transfer matrix representation for periodic planar media

A. Parrinello*, G.L. Ghiringhelli

Dipartimento di Scienze e Tecnologie Aerospaziali, Politecnico di Milano, Via La Masa 34, 20156 Milano, Italy

ARTICLE INFO

Article history:
Received 7 April 2015
Received in revised form
15 January 2016
Accepted 4 February 2016
Handling Editor: J. Astley
Available online 22 February 2016

Keywords:
FEM
Periodic Media
Transfer Matrix
Sound Transmission

ABSTRACT

Sound transmission through infinite planar media characterized by in-plane periodicity is faced by exploiting the free wave propagation on the related unit cells. An appropriate through-thickness transfer matrix, relating a proper set of variables describing the acoustic field at the two external surfaces of the medium, is derived by manipulating the dynamic stiffness matrix related to a finite element model of the unit cell. The adoption of finite element models avoids analytical modeling or the simplification on geometry or materials. The obtained matrix is then used in a transfer matrix method context, making it possible to combine the periodic medium with layers of different nature and to treat both hard-wall and semi-infinite fluid termination conditions. A finite sequence of identical sub-layers through the thickness of the medium can be handled within the transfer matrix method, significantly decreasing the computational burden. Transfer matrices obtained by means of the proposed method are compared with analytical or equivalent models, in terms of sound transmission through barriers of different nature.

1. Introduction

The design of sound barriers is of utmost importance in many applications including automotive, aerospace and buildings. Multi-layer panels are commonly used as wall partitions, airplane flooring and cabin structures and sound transmission through these components has great relevance. Sound transmission through panels can be evaluated experimentally [1], numerically or analytically. Since an alternative numerical approach is proposed here, a brief overview of the most common numerical methods is presented in this section.

Models based on well-known numerical methods, such as the Finite Element Method (FEM) [2–5] and the Boundary Element Method (BEM) [6], can provide an accurate computation of sound transmission. However, they require extensive computing resources and are inappropriate for large structures and high frequency calculation, when the vibration wavelength becomes much smaller than the structural dimensions.

Statistical Energy Analysis (SEA) [7,8] has also proven to be a useful tool in the task of predicting sound transmission through partitions. SEA is appropriate for high frequency calculation but is known to fail at low frequencies where the number of modal resonance frequencies in the analysis band is low. The SEA methodology can be exploited either with a *modal approach*, i.e. by modeling each subsystem as a superposition of the resonant responses, or with a *wave approach*, i.e. by modeling each subsystem as a superposition of waves traveling through the structure. The latter way consists in deriving and solving a dispersion set of equations between wavenumbers and frequencies for the subsystem of interest. Modal density, group and phase velocities, radiation efficiency and loss-factor are calculated using dispersion relations' solutions.

* Corresponding author.
E-mail addresses: andrea.parrinello@polimi.it (A. Parrinello), gianluca.ghiringhelli@polimi.it (G.L. Ghiringhelli).

http://dx.doi.org/10.1016/j.jsv.2016.02.005
0022-460X/

These quantities can be evaluated over many kinds of structures, e.g. for waveguides [9], curved laminates and composite sandwich panels [10]. The combination of a wave approach with the finite element (FE) model of a unit cell leads to the dispersion problem of the related periodic structure [11,12]. Vibroacoustic responses of various periodic structures are presented in [13,14]. The mathematics of wave propagation in periodic systems has been discussed by Brillouin [15] in the field of electrical engineering. Cremer and Leilich [16] and Heckl [17] investigate on periodic structures formed by assemblages of beams and plates. The effect of damping, the nature of propagation waves and their possible interaction with acoustic waves have been discussed by Mead [18,19]. In all the papers referred to, exact, harmonic solutions have been found for the equations of motion of the periodic system. Mead [20] and Abrahamson [21] have involved the Rayleigh–Ritz technique in order to treat non-uniform periodic structures. Afterwards, Orris and Petyt [22,23] have employed the FE technique for wave propagation analysis.

An alternative approach is the Transfer Matrix Method [24] (TMM). Matrix representation of sound propagation is an efficient and largely used tool for modeling plane acoustic fields in stratified media. The problem is formulated in the frequency domain. The layers are assumed to be laterally infinite and can be of different natures. Nonetheless, at low frequencies, where the effects of size are important, it is essential to include appropriate corrections, accounting for the finite radiating area. An approach, to the specific problem of airborne transmission losses, is based on application of the spatial windowing technique [10,25,26]. Analytical expressions for the transfer matrices are only available for elastic solids, thin plates, fluids and poro-elastic media. On the basis of the three-dimensional (3D) elasticity theory, Huang and Nutt [27] derive the transfer matrix of a general anisotropic layer. Description of non-homogeneous structures is difficult, e.g. honeycomb panels, ribbed panels, stud based double-leaf walls, panels with PZT patches and viscoelastic inclusions or functionally graded components in general. An equivalent homogeneous representation can be derived for some heterogeneous structures, such as for honeycomb panels. For other structures, homogenization may be ineffective. Moreover, significant contributions to the dynamic behavior could be lost in homogenization, especially local high-frequency dynamics.

The present work aims to extend the use of the TMM to more general barriers, e.g. the ones above mentioned, by exploiting the features of FE modeling. A procedure is derived to obtain the transfer matrix, related to a specific incident plane wave, for an infinite planar medium characterized by in-plane periodicity. First, a proper 3D unit cell of the periodic medium is modeled by means of FEs and the related dynamic stiffness matrix (DSM) is obtained. Then, this latter is manipulated by applying proper conditions to the periodic boundaries, according to the trace of the incident plane wave. Finally, a further condensation of the DSM, with respect to the trace of the incident plane wave, leads to the desired one-dimensional through-thickness model and the related transfer matrix.

The proposed procedure combines the versatility of a FE model and the efficiency of the TMM. The combination of a FE model of the cell with a wave approach accurately describes the dynamics involved in acoustic transmission, and the transfer matrix obtained makes it possible to exploit the ability of the TMM by efficiently combining layers of different natures, leading to simple and effective computation of all the required acoustic indicators. Moreover, matrix representation prevents the troubles associated with dealing with dispersion curves and SEA models, thereby avoiding the need for analyst intervention. Thus, the proposed procedure for evaluating the acoustic properties of a barrier can be termed as *direct*, since it implicitly involves the wave dispersion in the medium through the DSM of the related unit cell. In contrast, a procedure which requires the dispersion solution for the wave characterization of the medium could be termed as *indirect*. Possible applications of the proposed procedure on heterogeneous media include functionally graded plates [28], 3D braided composite [29], 3D woven composites [30,31] and stud based double-leaf walls [4,5].

Section 2 presents an overview of the TMM for the phenomenon of acoustic transmission through flat and infinitely extended media. The transfer matrix of a periodic medium will then be derived in Section 3, and a number of applications will demonstrate the effectiveness of the proposed procedure in Section 4.

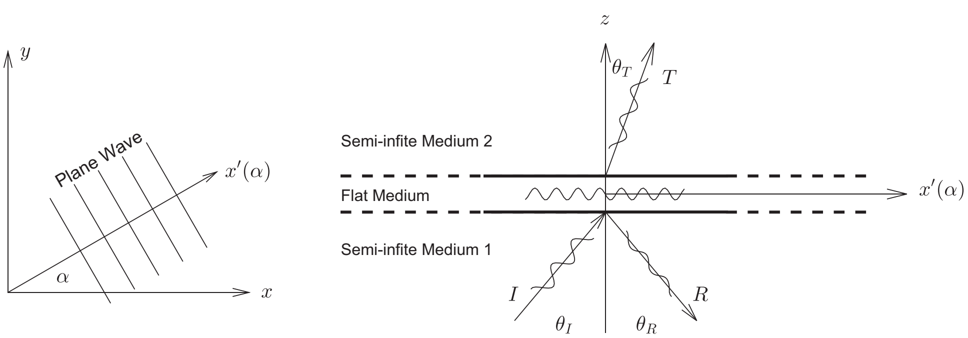

Fig. 1. Plane wave reflection and transmission at a plane interface between two semi-infinite media.