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Spontaneous rotation in a driven mechanical system

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ABSTRACT

We show that a mass free to circulate around a shaken pivot point exhibits resonance-like effects and large amplitude dynamics even though there is no natural frequency in the system, simply through driving under geometrical constraint. We find that synchronization between force and mass occurs over a wide range of forcing amplitudes and frequencies, even when the forcing axis is dynamically, and randomly, changed. Above a critical driving amplitude the mass will spontaneously rotate, with a fractal boundary dividing clockwise and anti-clockwise rotations. We show that this has significant implications for energy harvesting, with large output power over a wide frequency range. We examine also the effect of driving symmetry on the resultant dynamics, and show that if the shaking is circular the motion becomes constrained, whereas for anharmonic rectilinear shaking the dynamics may become chaotic, with the system mimicking that of the kicked rotor.

1. Introduction

Nonlinear oscillators with a characteristic frequency response, such as the canonical example of the damped, driven nonlinear pendulum, have been central in exploring dynamical effects such as resonance (see e.g. [1]), chaos [2], and fractal basins of attraction [3]. Somewhat surprisingly, the nature, and even possibility, of similar effects in oscillator systems with no characteristic frequency have received much less attention. The horizontal pendulum is the paradigmatic example of such a system, however work has focused primarily on the small amplitude limit, relevant to precision seismometry (see e.g. [4]). At the other extreme in dynamic response is the kicked rotor [5], well-known for its chaotic dynamics, physically realized with cold atoms [6] and even using Bose–Einstein condensates [7]. There has however been a recent surge of interest in rotating masses for energy harvesting [8–11], as these systems do not suffer many of the problems facing resonant energy harvesters [12,8], although in the context of sea wave energy harvesting, at least, there are a number of proposals for overcoming these limitations (see e.g. [13]).

Parallel to work on mechanical models there has been significant interest in recent years in the dynamics of particles in time-dependent external potentials. A primary focus has been on the symmetry-breaking required to observe ratchet-like energy transport [14], and similar effects have been observed for effective particles (self-localized solitons) in nonlinear wave systems [15,16]. The horizontally shaken pendulum considered here can be regarded as a particle in a time-dependent flashing periodic potential, but with only one period, supplemented by periodic boundary conditions. In this context we identify the conditions for particle motion, and describe the full bifurcation picture of the particle dynamics. Despite an enormous amount of interest in so-called flashing or pulsating ratchets [17,18], involving for instance a biharmonic time-varying potential [14], there appears to be no detailed study of the simpler case proposed in this work.

We begin in Section 2 by developing the time-dependent model of our system, including connection to the physical parameters natural in a horizontally shaken pendulum system. In Section 3 we consider the special case of purely rectilinear

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harmonic shaking and examine the stability of the fixed points and the periodic families, and uncover a number of bifurcations which occur as the shaking amplitude is varied. In Section 4 we examine the effects of more general types of driving, such as anharmonic rectilinear shaking, random changes in shaking direction, and elliptical and circular shaking. In Section 5 we then look at possible application of this system for energy harvesting, by examining characteristic energy dissipation over a range of parameter values. Finally in Section 6 we present our conclusions and directions for further work.

2. Model

We consider a mass M attached by a massless rod of length R to a driven pivot point, constrained to lie in the xy plane, as shown in Fig. 1(a and b). Gravity acts in the z direction, and we assume plays no role in the dynamics. To be consistent with the standard pendulum literature, we define $\theta = 0$ to correspond to the mass on the negative y-axis. The position of the mass is given by $x(t) = R\sin(\theta) + F_x(t)$ and $y(t) = R\cos(\theta) + F_y(t)$, where $F_i(t)$ denotes the pivot motion in the $i = x, y$ directions. There is no potential energy in the system, so the Lagrangian takes the particularly simple form $\mathcal{L} = T = \frac{1}{2}M\left(\dot{x}^2 + \dot{y}^2\right)$. Including dissipation through a Rayleigh function (see e.g. [1]), we find the following equation of motion for the dissipative, driven system:

$$\ddot{\theta} + \frac{\Gamma}{MR^2}\dot{\theta} + \frac{\ddot{F}_x}{R}\cos(\theta) - \frac{\ddot{F}_y}{R}\sin(\theta) = 0, \tag{1}$$

where we have assumed that the damping is occurring at the hub, and captures the effect of energy harvesting mediated by the coefficient Γ [19]. We will consider the effect of both harmonic and anharmonic shaking, with a focus on the former, where we assume the following specific forms: $F_x(t) = F\cos(\omega t)\cos(\phi)$, $F_y(t) = F\cos(\omega t)\sin(\phi)$ for rectilinear shaking; and $F_x(t) = F(f_x/(f_x + f_y))\cos(\omega t)$, $F_y(t) = F(f_y/(f_x + f_y))\sin(\omega t)$ for elliptical shaking. After rescaling time through $t = \tilde{t}/\omega$ the

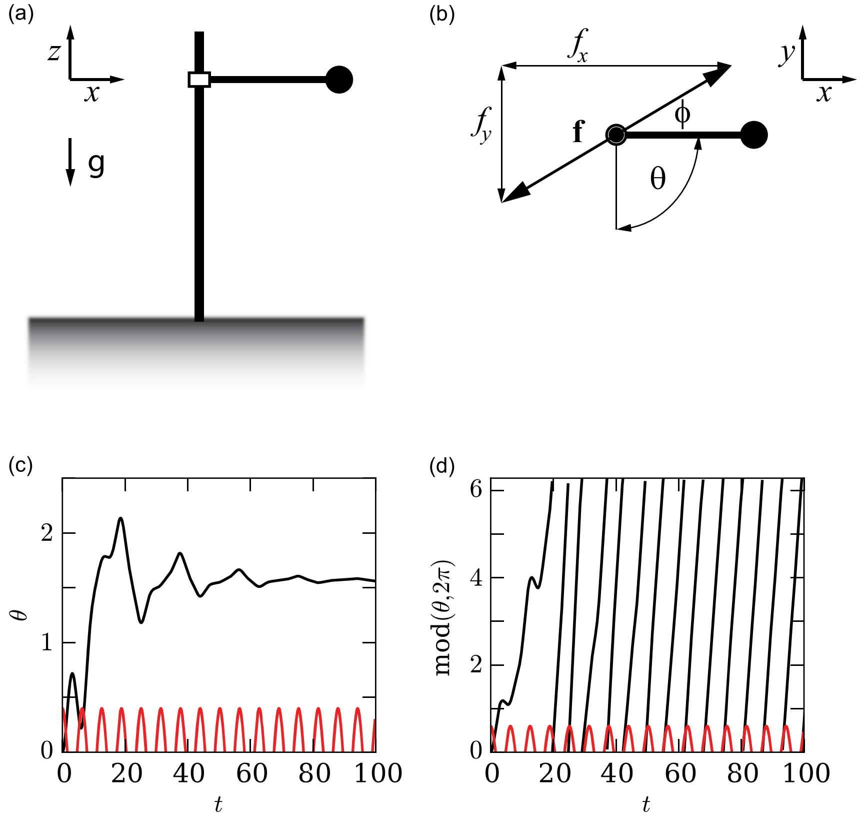

Fig. 1. (a) Schematic of a horizontally oriented pendulum, consisting of a mass supported by a massless rod, attached to a central rod which is shaken periodically; (b) top view of the system, showing the coordinate system in use, and the rectilinear shaking of the pivot point at angle ϕ; (c and d) examples of the dynamics resulting from harmonic shaking in the x-direction only with (c) convergence to a fixed point when f=0.4 and (d) convergence to a rotating solution when f=0.6. In both cases $\theta(0) = \dot{\theta}(0) = 0$ and $\gamma = 0.1$. The lines at the bottom (red) show the shaking oscillations for reference. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)