Journal of Sound and Vibration 371 (2016) 305–320

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Analytical solutions for a mass moving along a finite stretched string with random surface irregularities

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ARTICLE INFO

Article history:
Received 14 December 2015
Received in revised form
20 January 2016
Accepted 4 February 2016
Handling Editor: L.G. Tham
Available online 3 March 2016

Keywords:
Moving mass
surface irregularities
random responce
analytical solution

ABSTRACT

This paper derives the analytical solution for the stochastic analysis of a concentrated mass moving at a constant velocity along a finite stretched string with random surface irregularities. Firstly, the problem of computing the spectral power density (PSD) of the random response is transformed into the problem of computing the transient response of a concentrated mass moving at a constant velocity along a stretched string with harmonic random surface irregularities. Next, the analytical solutions of the contact force between the string and the mass are derived for harmonically varying surface irregularities. Finally, the PSDs of contact force and the displacement of the string are determined in terms of the PSD of the random surface irregularities. The analytical solutions cover all three cases of subsonic, sonic or supersonic velocities. The proposed method was verified based on a comparison with the semi-analytical method.

1. Introduction

The interaction between a moving force (or mass) and a structure has been comprehensively investigated [1,2], including investigations on a string subjected to a moving mass. The string model can be applied to simulate engineering problems, such as the contact wire in a coupled pantograph–catenary system [3] and some aerospace structures [4]. Analytical or numerical methods can be used to obtain the dynamic responses of a string subjected to a moving force or mass.

Kanninen and Florence [5], and Langlet et al. [6], derived the analytical solutions for a stretched infinite string subjected to a moving force at a constant speed. Studies also investigated a stretched infinite string excited by a moving force with a varying speed [7,8]. Wolfert et al. investigated the dynamic response of an non-homogeneous, stretched infinite string on an elastic foundation subjected to a moving force at a constant speed [9]. Dieterman and Kononov studied the dynamic response of a stretched infinite string on an elastically supported membrane subjected to a moving force [10]. Sagartz and Forrestal considered a stretched semi-infinite string subjected to a moving force with a constant acceleration [11]. Oniszczuk [12] and Rusin and Sniady investigated the dynamic response of a stretched finite double string connected to a Winkler elastic layer and subjected to a moving harmonic force at a constant speed [13].

Kruse et al. [14] obtained the eigenfrequencies of a stretched infinite string on a viscoelastic foundation subjected to a two-mass oscillator at a constant speed. Gao et al. [15] presented an exact solution for a point mass moving along a stretched infinite string on a Winkler foundation. Rodeman et al. [4] derived a numerical solution for a stretched

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http://dx.doi.org/10.1016/j.jsv.2016.02.006
0022-460X/

semi-infinite string subjected to a constantly accelerating moving mass. Smith [16] neglected the mass of the string and obtained an analytical solution for a stretched finite string subjected to a mass moving at a constant speed. Yang et al. [17] proposed an integral equation describing the response of a stretched finite string subjected to a moving mass at a constant speed, and then solved this equation through a numerical integration method. Lee et al. [18] adopted the Lagrange multiplier method to describe the dynamic responses of a coupled moving mass-stretched beam with a separation between each mass. The semi-analytical method [19] and the space-time method [20] were proposed to solve the problem of a stretched finite string subjected to a moving mass at a constant speed. Gao et al. [21] derived the analytical solutions for a stretched string subjected to a concentrated mass moving at a constant velocity, and the solutions covered an infinite, semi-infinite or finite string subjected to a moving mass at subsonic, sonic or supersonic velocities. Several numerical methods and semi-analytical solutions were proposed to solve the problems of the vibrations of structures subjected to moving loads or masses [22].

The above-mentioned studies dealt with the deterministic analysis of a string subjected to a moving force or a moving mass. However, in reality, the loads or the parameters of the structures are usually random. Extant literature has examined different aspects of uncertainties for moving force or moving mass problems. Frýba [23] investigated the non-stationary random vibration of a beam subjected to a random force with constant mean value, moving at a constant speed. Lwankiewicz and Śniady [24] considered the random dynamic response of' a beam subjected to concentrated forces with random amplitudes and developed an analytical technique to determine the response of the beam. Yoshimura et al. used the Galerkin finite element method, the linearization technique and the implicit direct integration method, [25] to investigate the longitudinal and transverse deflections of nonlinear beams with random irregularities subjected to a moving load. Ricciardi [26] considered the random response of a beam excited by moving loads with random amplitudes, and provided a closed-form solution of the non-stationary stochastic response. Frýba et al. [27] investigated the random response of an infinitely long beam resting on an elastic random foundation, subjected to a constant force moving with a constant speed. Zibdeh and Rackwitz [28] considered the higher order moments of a simply supported elastic beam subjected to a stream of random loads moving with a time varying velocity. Based on Euler–Bernoulli beam theory and stochastic methods, Zibdeh [29] derived the closed-form solutions for the mean and variance for the random vibration of a simply supported elastic beam subjected to random loads moving with time-varying velocity. Śniady et al. [30] obtained an analytical expression for calculating the probabilistic characteristics of the beam response to a load moving with stochastic velocity based on its integration and differentiation rules. Au et al. [31] considered the vibration of cable-stayed bridges under moving railway trains by taking the random rail irregularities and the geometric nonlinear behavior of the cable-stayed bridge into account. Andersen et al. [32] investigated the random response of a single-degree-of-freedom vehicle moving at a constant velocity along an infinite Bernoulli–Euler beam with surface irregularities, which was supported by a Kelvin foundation. Andersen and Nielsen [33] proposed a first-order perturbation method to address the stochastic analysis of a single-degree-of-freedom vehicle moving at constant velocity along an infinite Bernoulli–Euler beam resting on a Kelvin foundation with random support stiffness. Lu et al. [34] used the pseudo excitation method (PEM), and considered the non-stationary random vibration of vehicle–bridge systems, and adopted a vehicle–bridge interaction element to reduce the computational effort.

This study derives an analytical solution for the stochastic response of a concentrated mass moving at a constant velocity along a finite stretched string with random surface irregularities. The analytical solutions cover all three cases, namely, subsonic velocity, sonic velocity and supersonic velocity. The proposed method was verified based on a comparison with the semi-analytical method.

2. The governing equation

In this study, a finite stretched string with random surface irregularities subjected to a concentrated mass M moving at a constant speed V is considered. The string is clamped at both ends and has the length L, line mass density ρ and tensile force T. The string axis is supposed to form a straight line in the state of static equilibrium. The surface of the string has the random irregularities, $R(x)$, measured from the mean level of the surface. The mean level of the surface is assumed to be unchanged along the string and is parallel to the string axis. The mass moves from left to right, starting from the left end of the string. Let $Y(t)$ and $W(x,t)$ be the vertical displacements of the mass and the string relative to the mean level of the surface, respectively. Furthermore, the contact force between the mass and the string is denoted by $P(t)$, as shown in Fig. 1.

The system is linear, and the irregularities are assumed to be random processes with zero mean. Hence, the system response can be expressed as a random process with a deterministic mean value and a random centered value. Moreover, as the gravitational force of the mass is deterministic, it is easy to prove that the deterministic mean value is the response of the string with no irregularities subjected to a moving mass due to the gravitational force of the mass. The random centered value is the response of the string with irregularities and subjected to a moving mass, neglecting the force of gravity. The authors derived the solution for the problem of the string with no irregularities, subjected to a moving mass due to the force of gravity of the mass in a previous study [21]. Hence, the focus of this paper is the random centered value of the response.