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On the shielding of sound from a source near one or two coaxial cones

L.M.B.C. Campos *, P.J.S. Gil

CCTAE, IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal

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ABSTRACT

The shielding of the sound from a source near a cone or two coaxial circular cones is addressed by an exact analytical method, considering free spherical waves satisfying a rigid or impedance boundary condition on the wall of the cone(s); the boundary condition is applied for impedance proportional to the distance from the vertex. The exact solution involves the eigenvalues of the problem that are generally real or complex (not integers), and coincide with the degree of the associated Legendre functions and order of the spherical Bessel functions which specify respectively the latitudinal and radial dependencies of the wave field. These eigenfunctions or 'conical wave harmonics' can be chosen in more than one way, e.g. (a) as standing wave modes which are finite at the vertex of the cone(s), but do not satisfy a radiation condition at infinity; (b) as propagating waves satisfying a radiation condition at infinity, and generally singular at the vertex of the cone (s). The method of calculation of eigenvalues and eigenfunctions is presented both for a single cone and two coaxial cones with the same vertex, and arbitrary aperture(s). The method specifies the eigenvalues, and the corresponding radial, azimuthal and latitudinal eigenfunctions. An asymptotic formula is obtained for the eigenvalues which gives reasonable good agreement with the exact results. The eigenvalues and eigenfunctions appear with suitable amplitudes in the Green function representing a monopole source near the vertex of the cone(s). The acoustic field is plotted also for a longitudinal and a transverse dipole and mixed quadrupole source near the vertex.

1. Introduction

The simplest description of sound propagation in a duct of varying cross-section is the Webster [1] horn equation, actually derived before by Rayleigh [2] that applies to quasi-one-dimensional propagation, that is for wavelengths larger than the cross-section; it can be extended to nozzles of varying cross-section, e.g. at low Mach number [3]. The mean flow velocity varies with the inverse of the cross-section to conserve the volume flux. In the ray approximation of slow variation of cross-section on a wavelength scale, the amplitude varies with the inverse square root of the cross-section [4]. In the case of a nozzle, the slow variation of the mean flow velocity leads to a phase shift specified by the integrated Doppler factor [5]. The horn wave equation has exact solutions in terms of elementary functions [6] for five shapes: (i) exponential [7]; (ii) hyperbolic cosine [8] or secant; (iii) circular sine [9] or cosecant. For the low Mach number nozzles with the same shapes the exact solutions involve special functions: (i) confluent hypergeometric functions for the exponential nozzle [10]; (ii)–(iii)

* Corresponding author. Tel.: +351 218417267.
E-mail addresses: lmbcampos.aero@ist.utl.pt (L.M.B.C. Campos), p.gil@dem.ist.utl.pt (P.J.S. Gil).

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modified Mathieu functions for catenoidal, sinusoidal and inverse nozzles [11,12]. The quasi-one-dimensional propagation of sound in horns used in loudspeakers subject to viscous dissipation is analogous to the longitudinal vibrations of visco-elastic rods [13] used in power tools. The assumption of quasi-one-dimensional propagation applies if the cross-section does not become too large, e.g. for a cylindrical duct with sinusoidal wall undulations [14,15]. In the case of power-law horns [16,17], including the conical case, the quasi-one-dimensional approximation is valid only near the vertex, and otherwise transverse wave modes must be considered.

In the case of a conical duct or two coaxial cones, the sound source may be placed at the vertex leading to two problems: (i) the 'interior' problem of sound propagation in a cone $0 < \theta_i < \alpha$ with aperture $\alpha < \pi/2$ (point I in Fig. 1) or between two cones $\alpha_1 < \theta_i < \alpha_2$ with the same axis and apertures $0 < \alpha_1 < \alpha_2 < \pi/2$ (point I in Fig. 2); (ii) the 'exterior' problem of shielding of a point sound source by a cone $\alpha < \theta_e < \pi$ with aperture $\alpha > \pi/2$ (point E in Fig. 1) or two cones $\alpha_1 < \theta_e < \alpha_2$ with opposite axes and apertures $\alpha_1 < \pi/2 < \alpha_2$ (point E in Fig. 3).

The problem of reflection or shielding of sound by obstacles is of major interest in many situations like building acoustics, traffic noise, noise barriers, or 'installation factors' in vehicle design. For example, in aircraft design, the engine position can affect noise, either negatively by reflection, which may reinforce sound, or positively by interference or shielding effects, which reduce noise levels. The noise reinforcement or shielding depends on the location of the source and observer relative to the 'obstacle'. The need to consider 'obstacles' with complex shapes (buildings, cars, airplanes) leads to the use of approximate methods, restricted to low-frequency scattering [18,19] or high-frequencies sound rays [20–22], as well as intermediate approximations, such as Fresnel methods [23,24]. The present paper obtains exact solutions for both the sound propagation and noise shielding problems for a sound source near the vertex of one or two coaxial cones.

The present approach to the problem of sound propagation in or shielding by a cone or coaxial cones (Section 1) is to use the 'conical wave harmonics' (Section 2). These are spherical harmonics (Section 2.1) satisfying a rigid body (or linear impedance wall) boundary condition at the surface of one cone (Section 2.2, Fig. 1) or two coaxial cones with the same vertex (Section 2.3). It can be shown that the eigenvalues are the order of the spherical Bessel functions (and degree of the associated Legendre functions), and are generally real (or complex), but not integers (Section 2.2). It follows that the conical wave harmonics are specified by associated Legendre functions (Section 2.3), rather than the polynomials which apply to spherical harmonics. The eigenfunctions (Section 3) corresponding to the eigenvalues for a single cone or two cones (Section 3.1) may be chosen as standing modes finite at the vertex (Section 3.2) or propagating waves satisfying a radiation condition at infinity (Section 3.3). The eigenvalues can be calculated (Section 4) by a self-checking recurrence method (Section 4.1) and specify also the eigenfunctions (Section 4.2); asymptotic approximations are obtained for the eigenvalues and eigenfunctions (Section 4.3).

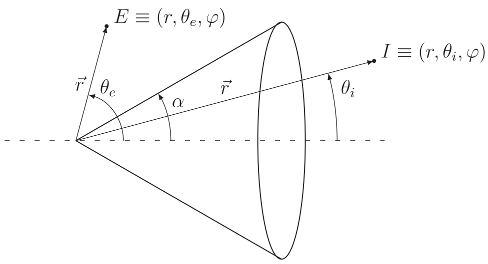

Fig. 1. Cone of aperture α with an observer inside at position $I = (r, \theta_i, \varphi)$. When $\alpha > \pi/2$, the cone can be thought as being oriented in the opposite direction with aperture $\pi - \alpha$ and the observer at $E = (r, \theta_e, \varphi)$ in the outside.

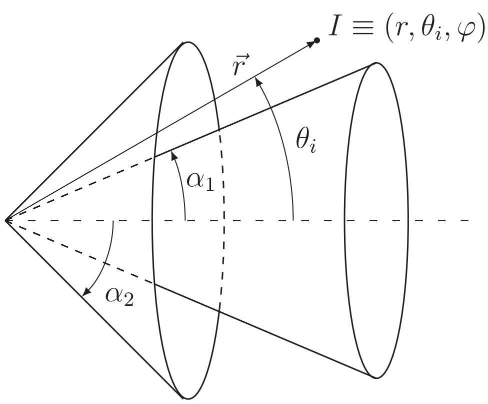

Fig. 2. Observer between two cones with the same vertex and axis.