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An alternative approach to measure similarity between two deterministic transient signals

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ABSTRACT

In many practical engineering applications, it is often required to measure the similarity of two signals to gain insight into the conditions of a system. For example, an application that monitors machinery can regularly measure the signal of the vibration and compare it to a healthy reference signal in order to monitor whether or not any fault symptom is developing. Also in modal analysis, a frequency response function (FRF) from a finite element model (FEM) is often compared with an FRF from experimental modal analysis. Many different similarity measures are applicable in such cases, and correlation-based similarity measures may be most frequently used among these such as in the case where the correlation coefficient in the time domain and the frequency response assurance criterion (FRAC) in the frequency domain are used. Although correlation-based similarity measures may be particularly useful for random signals because they are based on probability and statistics, we frequently deal with signals that are largely deterministic and transient. Thus, it may be useful to develop another similarity measure that takes the characteristics of the deterministic transient signal properly into account. In this paper, an alternative approach to measure the similarity between two deterministic transient signals is proposed. This newly proposed similarity measure is based on the fictitious system frequency response function, and it consists of the magnitude similarity and the shape similarity. Finally, a few examples are presented to demonstrate the use of the proposed similarity measure.

1. Introduction

In many practical engineering situations, the degree of similarity between two signals is required to examine the system and to make an appropriate decision. Typical examples consist of pattern recognition problems, such as classification, clustering, and retrieval problems [1–4]. Other cases include machinery condition monitoring where, for example, a measured vibration signal is compared with a healthy reference signal to monitor whether or not any fault symptom is developing [5,6], structural damage detection where the model of damaged structure is compared with an undamaged model [7,8], and modal analysis where the degree of correlation between a finite element model (FEM) and the measured frequency response function is examined [9–11].

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Various types of similarity measures can be used in such engineering applications, and many of these consist of either distance-based measures (such as the Euclidean distance, the Chebyshev distance, rank distance, etc.) or correlation-based measures (such as the correlation coefficient, Pearson correlation, Spearman correlation, etc.) [1,12–14]. Also, dynamic time warping (DTW) is extensively used to compare two time signals, especially for speech signals [4,15]. In addition, many other similarity measures have been developed for specific applications with specific requirement. For example, a combination of several existing similarity measures have been suggested to improve classification results [16,17], image processing and pattern recognition can be combined to identify subtle differences in the vibration mode shapes [18], and the modal similarity index has been proposed to measure the reliability of vibration modes [19].

Of the various similarity measures mentioned above, the measure that is most frequently used for time signals in sound and vibration applications is the correlation coefficient, which is defined as

$$\rho_{xy} = \frac{\mathrm{Cov}(x, y)}{\sigma_x \sigma_y} \tag{1}$$

where Cov(x, y) is the covariance between signals $x(t)$ and $y(t)$, and σ_x and σ_y are the standard deviations of the corresponding signals. The correlation coefficient can be interpreted as a measure of the linear relationship between two signals. That is, $\rho_{xy} = 0$ if two signals have no linear relationship, and $|\rho_{xy}| = 1$ if two signals are perfectly matched in a linear manner. In the frequency domain, the frequency response assurance criterion (FRAC) is extensively used, especially for modal analysis [7–11]. This can be interpreted as the frequency domain equivalence of the correlation coefficient and it is defined as

$$\mathrm{FRAC} = \frac{\left| \sum_{f=f_1}^{f_2} X(f) Y^*(f) \right|^2}{\sum_{f=f_1}^{f_2} X(f) X^*(f) \sum_{f=f_1}^{f_2} Y(f) Y^*(f)} \tag{2}$$

where f_1 and f_2 are the lower and upper frequency limits respectively, $X(f)$ and $Y(f)$ are the frequency data (usually frequency response functions), and the symbol '$*$' denotes the complex conjugate. The FRAC is also a measure of the linear relationship between two frequency data, i.e., it is zero if there is no linear relationship between two data, and it is unity if two frequency data are completely linearly related.

These correlation-based similarity measures were developed as probabilistic and statistical measures, and have been successfully applied in numerous applications involving sound and vibration. However, in some cases with deterministic transient signals, researchers have found that correlation-based measures do not always provide sensible results. This may be a result of the correlation-based similarity measures taking into account both the magnitude and shape of signals simultaneously, without properly reflecting the characteristics of the deterministic transient signal.

Recently, the group delay of a fictitious system frequency response function was introduced as an alternative measure for the shape similarity of a deterministic transient signal [20]. In this paper, the measurement of the similarity between two deterministic transient signals is generalized according to the fictitious system frequency response function by separately considering the magnitude similarity and the shape similarity. This newly-proposed similarity measure is referred to as the "magnitude–shape (*M–S*) similarity measure." The *M–S* similarity measure has two types of usage, one is an M-S similarity function that is a function of the frequency, and the other is the *M–S* similarity index that is a single numbered index that is similar to most existing similarity measures. Both consist of the magnitude similarity and shape similarity, and it is particularly noted that the *M–S* similarity function can be used like a coherence function, that is, frequency components that are more similar (or dissimilar) than others can be found. The next section describes the details of the *M–S* similarity measure, and then examples of the use of the new measure are presented to show that a better interpretation can be achieved by separately examining the magnitude similarity and shape similarity.

2. The *M–S* similarity measure based on the fictitious system FRF

Suppose we have two real-valued deterministic transient time signals, $x(t)$ and $y(t)$. Since these are deterministic, they possess unique representations in the frequency domain, i.e., $X(f)$ and $Y(f)$, respectively, as below

$$X(f) = |X(f)| e^{j\phi_x(f)} \text{ and } Y(f) = |Y(f)| e^{j\phi_y(f)} \tag{3}$$

where $|X(f)|$ and $|Y(f)|$ are the magnitude spectra, and $\phi_x(f)$ and $\phi_y(f)$ are the phase spectra. It is noted that these spectra are continuous in the frequency domain since the time signals are transient. Even if there are no physical input–output relations

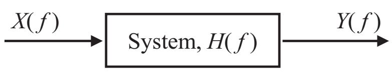

Fig. 1. A fictitious linear time-invariant system.