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A mapping relationship based near-field acoustic holography with spherical fundamental solutions for Helmholtz equation

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A B S T R A C T

In the procedure of the near-field acoustic holography (NAH) based on the fundamental solutions for Helmholtz equation (FS), the number of FS and the measurement setup to obtain their coefficients are two crucial issues to the successful reconstruction. The current work is motivated to develop a framework for the NAH which supplies a guideline to the determination of the number of FS as well as an optimized measurement setup. A mapping relationship between modes on surfaces of boundary and hologram is analytically derived by adopting the modes as FS in spherical coordinates. Thus, reconstruction is converted to obtain the coefficients of participant modes on holograms. In addition, an integral identity is firstly to be derived for the modes on convex surfaces, which is useful in determining the inefficient or evanescent modes for acoustic radiation in free space. To determine the number of FS adopted in the mapping relationship based NAH (MRS-based NAH), two approaches are proposed to supply reasonable estimations with criteria of point-wise pressure and energy, respectively. A technique to approximate a specific degree of mode on patches by a set of locally orthogonal patterns is explored for three widely used holograms, such as planar, cylindrical and spherical holograms, which results in an automatic determinations of the number and position of experimental setup for a given tolerance. Numerical examples are set up to validate the theory and techniques in the MRS-based NAH. Reconstructions of a cubic model demonstrate the potential of the proposed method for regular models even with corners and shapers. Worse results for the elongated cylinder with two spherical caps reveal the deficiency of the MRS-based NAH for irregular models which is largely due to the adopted modes are FS in spherical coordinates. The NAH framework pursued in the current work provides a new insight to the reconstruction procedure based on the FS in spherical coordinates.

1. Introduction

To locate the position and target the strength of noise for a vibrating structure, near-field acoustic holography (NAH) had been widely adopted as an effective tool. It has a significant influence on the noise diagnostics, which gives a permission to get all desired acoustic quantities, such as pressure, particle velocity, sound power, etc. from a number of discrete field measurement.

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It was originally developed by Willams, Manynard, etc. to reconstruct surface velocity of a rectangular plane with Fourier transform technique [1–3]. Initially, the Fourier-based NAH decomposes the field pressure into *k*-space (wavenumber space) for baffled problems. In other words, the field pressure are expanded into plane waves and the reconstruction procedure is to obtain coefficients of the plane waves based on measured pressure. Although different from the *k*-space decomposition, concept of Fourier transformation was inherent to the 3D cylindrical and spherical NAH problems as the in-depth discussions in Ref. [4].

Since it was proposed [1], varieties of approaches had been proposed and their superiorities had been proven in various applications, which resulted in several categories according to their underlying theories. Statistical Optimal NAH [5–7] uses the elemental waves to approach the acoustic field, in which the surface-to-surface projection of the sound field is performed by using a transfer matrix defined in such a way that all propagating waves and a weighted set of evanescent waves are projected with optimal average accuracy [6]. Boundary element method (BEM)-based NAH [8–13] are appropriate for arbitrarily shaped model in which a general transformer matrix between the surfaces of structure and hologram is derived from the integral equation. Among the BEM-based NAH, two types of integration equation are adopted: the direct formulation (Helmholtz integral equation) and indirect formulation (single or double layer integral equation). The quantities reconstructed by the NAH derived from direct formulation have clear physical meaning [8–10], while the ones obtained by NAH derived from the indirect formulation are not the real physical quantities [11–13]. The equivalent source method (ESM) [14–19], also named as wave superposition algorithm (WSA) [16,20,21], was proposed by Koopman [22] for solving acoustic radiation problems of closed sources. ESM assumes the field is generated by a series of simple sources such as monopoles and dipoles, and numerical integration is not needed in determining the source strength for a set of prescribed positions. Despite versatility of the ESM and various successful applications, “retreat distance” between the actual source surface and the virtual source cannot be well defined and deserves more attention in the application [23]. The Helmholtz equation least square method (HELS) [24–26] adopted the spherical wave expansion theory to reconstruct acoustic pressure field from a vibrating structure. Coefficients of the spherical wave function, the fundamental solution for the Helmholtz equation (FS), are determined by requiring the assumed form of solution to satisfy the pressure boundary condition at the measurement points. Since the spherical wave functions solve the Helmholtz equation directly, it is immune to the nonuniqueness difficulty inherent in BEM-based NAH [27]. However, HELS works better for spherical or chunky model than elongated model due to the specific basis function [25].

Essentially speaking, NAH is to achieve the desired acoustic quantities by the measured physical quantities such as sound pressure in the field. Most of the methods explicitly require the transfer operator $\mathrm{T}(\mathbf{y},\mathbf{x})$ between desired acoustic quantities $f(\mathbf{y})$ and measured physical quantities $\overline{p}(\mathbf{x})$. They built a linear system of $f(\mathbf{y}) = inv(\mathrm{T}(\mathbf{y},\mathbf{x}))\overline{p}(\mathbf{x})$ in which $inv(\star)$ represents an inverse operator, by either a general numerical method (BEM-based NAH) [8–13], or specific basis spaces such as a general Fourier basis (Fourier-based NAH) [1–4], simplified monopoles, dipoles (ESM, WSA) [14–16,18,20,21], fundamental solutions (HELS) [24–27]. The reconstruction procedure is therefore to solve the linear system to obtain the physical quantities on the boundary, such as pressure or normal velocity in BEM-based NAH, the source strength of equivalent source in ESM, coefficients of basis functions in Fourier-based NAH and HELS, and following by an extrapolation process to achieve desired acoustic quantities.

Unfortunately, all the proposed methods are very sensitive to errors which may cause reconstruction to fail. It is primarily due to abundant adoption of basis functions in the transfer operator which amplifies the errors in the inverse process. That is the reason why there have been numerous studies focusing on the development of regularization methods to stabilize this inverse problem, such as truncated singular value decomposition [28] and the Tikhonov regularization [29]. Thus, construction of transfer operator is not a trivial process but is crucial to the feasibility and accuracy of the NAH. Concerning the theory development and practical measurement, it naturally arise a question whether there exists a guideline to determine the number of generalized basis function as well as measurement to obtain their coefficients for a given shape of source surface and prescribed tolerance.

To the best knowledge of authors, the number of FS as well as number and position of the microphones array in the measurement are not well studied for the category of NAH based on the FS. Thus, the primary objective of the current work is to build a guideline for the determination of the number of FS and measurement configuration in the FS based NAH by exploring the mapping relationship between the modes in FS between surface and hologram, and investigating approximation of the modes with a set of locally orthogonal patterns.

This paper is organized as follows: acoustic modeling with the boundary integral equation (BIE) and its multipole expansion form are briefly introduced in Section 2.1, two types of truncation techniques for the multipole BIE are derived in Section 2.2, which serves as fundamental theory for the determination of necessary degree of modes and also the explanation of the evanescent modes. Procedure of the MRS-based NAH as well as the evanescent waves and comparison with existing methods are described in Section 3. Section 4 is about the measurement configuration for three representative holograms. An algorithm to automatically determine the number and position of microphone array on planar hologram is introduced in part A, a reduced algorithm is developed for cylindrical holograms in part B, part C is about an analytical method for the determination of measurement configuration on spherical hologram. A number of numerical examples are elaborately designed to validate the theory and demonstrate the potential of the proposed MRS-based NAH in Section 5. Section 6 concludes the paper with some discussions and remarks of our method.