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On the dynamic stability of shear deformable beams under a tensile load

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A B S T R A C T

Loss of stability of beams in a linear static context due to the action of tensile loads has been disclosed only recently in the scientific literature. However, tensile instability in the dynamic regime has been only marginally covered. Several aspects concerning the role of shear deformation on the tensile dynamic instability on continuous and discontinuous beams are still to be addressed.

It may appear as a paradox, but also for the case of the universally studied Timoshenko beam model, despite its old origin, frequency-axial load diagrams in the range of negative values of the load (i.e. tensile load) has never been brought to light.

In this paper, for the first time, the influence of a conservative tensile axial loads on the dynamic behaviour of the Timoshenko model, according to the Haringx theory, is assessed. It is shown that, under increasing tensile loads, regions of positive/negative fundamental frequency variations can be distinguished. In addition, the beam undergoes eigenmode changes, from symmetric to anti-symmetric shapes, until tensile instability of divergence type is reached. As a further original contribution on the subject, taking advantage of a new closed form solution, it is shown that the same peculiarities are recovered for an axially loaded Euler–Bernoulli vibrating beam with multiple elastic sliders. This latter model can be considered as the discrete counterpart of the Timoshenko beam-column in which the internal sliders concentrate the shear deformation that in the Timoshenko model is continuously distributed. Original aspects regarding the evolution of the vibration frequencies and the relevant mode shapes with the tensile load value are highlighted.

1. Introduction

The study of the elastic stability of beams subjected to axial loads in a static context (columns) aims at the evaluation of the critical (buckling) load, that implies loss of uniqueness of the solution (bifurcation point), and the assessment of the instability shape assumed by the beam departing from its initial rectilinear configuration. Within the context of elastic stability, unstable configurations of straight beams have been historically identified under the action of compressive loads, however, the attention of the authors has been drawn by a recent research line underlining conditions for the instability of straight beams under tensile axial loads that stimulated the present work.

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When the stability of the column is studied in a vibrating beam (addressed as beam-column) it becomes crucial the assessment of the influence of the axial load on the natural vibration frequencies. The latter influence is obtained by solving the eigen-value frequency-axial load characteristic equation and it is usually given a graphical representation by means of the axial load-frequency curves. As the level of the axial load increases, if the characteristic curves intersect the axis at zero frequency, instability occurs with a progressive departure at zero frequency from the initial straight configuration, corresponding to no inertia forces, and it is denoted as divergence type instability [20].

In case the axial load is acting along the initial straight beam configuration and keeps constant its direction during vibration, it shows a conservative character and instability occurs with divergence.

In the study of linear elastic dynamic stability under axial conservative loads the influence of the shear deformation of the beam has been assessed in the literature by means of the well known Timoshenko beam-column model. In the latter case exhaustive stability analyses under compressive conservative axial loads only are available in the literature [1,10,20].

However, the role of the shear deformability in the vibration of Timoshenko beam-column when subjected to tensile axial load has never been studied. To the authors opinion, this is due the very recent disclosure of the instability phenomenon under tensile loads in the specific literature. This lack of investigation motivated the authors to provide the contribution presented in this work.

Static instability of shear deformable beams under tensile axial loads has been first shown in the literature by Kelly [17], who showed that tensile buckling is encountered in short beams such as multilayer elastomeric bearings. In the latter paper it is discussed that no tensile buckling occurs for high shear stiffness while tensile buckling load values are comparable to compressive buckling load values for low shear stiffness. However, in view of the possible cavitation phenomenon, the isolation device buckles only if the tensile critical load value is lower than the cavitation load. Tensile buckling has been also discussed for slender Timoshenko columns in presence of a weakened cross-section by Zapata-Medina et al. [22] and, furthermore, given more evidence by the authors in case of multiple cracked cross-sections [6]. In any case what emerges from the previous studies, entirely conducted in a static context, is that the questioned instability phenomenon seems to be governed by the shear deformability of the beam.

With regard to the Timoshenko model [20] it has to be pointed out that two different theories, attributed one to Engesser [12] and the other to Haringx [14–16], were presented. The former is based on the assumption that the shear force is orthogonal to the deformed beam axis, while the latter assumes that the shear force is tangent to the cross section in its deformed configuration. The formulation provided by Haringx, applied to compressed helical springs and rubber cylinders, has been recognised more accurate [20] and shown to provide more acceptable compressive critical loads and, furthermore, proved to be the only one able to predict tensile buckling loads [13,2].

For the latter reasons, since the intention of the authors is to explore the behaviour of shear deformable beams under the action of tensile loads, in this work the Haringx theory will be adopted to fill the gap existing in the literature regarding the study of the dynamic instability of the Timoshenko beam subjected to tensile axial load.

In the present paper, first, the dynamic behaviour of the Timoshenko beam-column subjected to an external conservative axial tip load is investigated. The frequency-axial load diagrams in the range of negative values of the load (i.e. tensile load) are brought to light for the first time.

The peculiar dynamic behaviour of the Timoshenko clamped–clamped beam-column under tensile loads, implying abrupt changes of the vibration mode, is discussed. In fact, as the tensile load increases, regions of positive or negative fundamental frequency variations can be distinguished before a divergence type tensile instability is reached.

Following a different reasoning, for the case of Euler–Bernoulli beams (i.e. no shear deformations are accounted for), with a single structural junction allowing transversal deflection discontinuities, Zaccaria et al. [21] showed that tensile instability of the divergent type occurs too. In addition, instability due to tensile loads has been shown to occur also as a results of the effects of constraint's curvature [3]. The link between the tensile buckling in Timoshenko beam and suitably deviced Euler–Bernoulli beams has been discussed by the authors who showed that, in the restricted static context, internal sliders allowing deflection discontinuities can be interpreted as concentrated shear deformations along the beam axis [9] if the internal sliders are endowed with internal translational springs. Namely the authors had shown that the Euler–Bernoulli beam with concentrated shear deformations can be regarded as the discrete counterpart of the Timoshenko beam.

Within the subject of tensile instability, the study by Zaccaria et al. [21] on the dynamic instability of Euler–Bernoulli beam with an internal slider gave certainly a significant contribution, however, at this stage, it requires a continuation to assess the influence of internal elastic springs and the effect due to the presence of an arbitrary number of internal elastic sliders.

To this purpose, since classical approaches available in the literature (enforcement of continuity conditions at the sliders positions or the transfer matrix approach) have been proved to be not convenient to pursue explicit closed form solutions, in this work the authors rather used a distributional approach, shown to be a parsimonious computational strategy capable of providing explicit closed form expressions of the solutions of discontinuous beams [7] or else efficacious finite element formulations of discontinuous beams and frames [8].

With the means of the distributional approach, in this work, investigation on dynamic instability due to conservative tensile loads is also conducted to study the effect of transversal displacement discontinuities along the span of a Euler–Bernoulli beam. Precisely, the exact closed-form solution of the beam-column, in presence of internal sliders, endowed with elastic springs, is derived. The study of Euler–Bernoulli beam with a single internal slider is not new since already treated in the literature by Zaccaria et al. [21], however the proposed solution represents an extension of the latter work to the case of