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Numerical study of acoustic instability in a partly lined flow duct using the full linearized Navier–Stokes equations

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Lined ducts are extensively applied to suppress noise emission from aero-engines and other turbomachines. The complex noise/flow interaction in a lined duct possibly leads to acoustic instability in certain conditions. To investigate the instability, the full linearized Navier–Stokes equations with eddy viscosity considered are solved in frequency domain using a Galerkin finite element method to compute the sound transmission in shear flow in the lined duct as well as the flow perturbation over the impedance wall. A good agreement between the numerical predictions and the published experimental results is obtained for the sound transmission, showing that a transmission peak occurs around the resonant frequency of the acoustic liner in the presence of shear flow. The eddy viscosity is an important influential factor that plays the roles of both providing destabilizing and making coupling between the acoustic and flow motions over the acoustic liner. Moreover, it is shown from the numerical investigation that the occurrence of the sound amplification and the magnitude of transmission coefficient are closely related to the realistic velocity profile, and we find it essential that the actual variation of the velocity profile in the axial direction over the liner surface be included in the computation. The simulation results of the periodic flow patterns possess the proper features of the convective instability over the liner, as observed in Marx et al.'s experiment. A quantitative comparison between numerical and experimental results of amplitude and phase of the instability is performed. The corresponding eigenvalues achieve great agreement.

1. Introduction

Acoustic liners are widely used to reduce noise emission of aero-engines and other turbomachines in both inlet and outlet ducts. In these cases, the interaction between sound and flow is complex because of the presence of grazing flow, especially at the vicinity of the wall. The function of the liner is to absorb the acoustic energy such that the sound wave continuously decays when propagating away from the source. However, in certain conditions the energy of mean flow can possibly be converted to acoustic energy, leading to sound amplification. This indicates the presence of a hydrodynamic instability over the liner, which is the major concern of the present investigation.

The hydrodynamic instability resulting from the sound/flow interaction over an impedance wall in a duct has been a challenging problem, both theoretically and experimentally. The solution of hydrodynamic instability was considered a a result of using the Ingard-Myers boundary condition [1,2] based on the assumption of an infinitely-thin vortex sheet over an

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impedance wall in previous works [3–5]. When viscous and nonlinear effects were neglected, the so-called unstable solution, once occurred, exhibited exponential growth in the form of a Kelvin–Helmholtz instability wave [6], which is ill-posed [7] and must be avoided. Various models have been proposed to modify the Ingard-Myers boundary conditions either by introducing a finite boundary layer thickness [8–10] or by changing the matching condition within the vortex sheet [11]. The theoretical analysis of Rienstra et al [8]. indicated that there was a critical boundary layer thickness δ_c, and the absolute instability would disappear when the boundary layer thickness was thicker than the critical value. Furthermore, in most situations, the real boundary layer was much thicker than the critical value δ_c, and this could explain why the instability wave was hardly observed.

However, the sound amplification in a lined duct has been discovered in recent laboratory experiments [12,13]. The experiments were conducted in the case of small-height/radius ducts with low-resistance liners and low duct-height/radius based Helmholtz numbers; and the sound amplification occurred when the averaged Mach number of the grazing flow was sufficiently large, i.e.$M > 0.2$. The details of acoustic and flow behaviors in a lined duct with grazing flow were measured [13], showing that the acoustic transmission coefficient had a minimum at the resonance frequency of the liner in the absence of flow, as expected; however, a strong increase in the transmission coefficient occurred near the resonance frequency as the Mach number of the grazing flow increased. Meanwhile, periodic flow pattern in the vicinity of the liner was visualized and the accurately-measured axial and transverse velocity fluctuations exhibited the features of an instability wave [13].

Several models have been set up to consider the effect of viscosity on the sound propagation within sheared flow [14–17]. The more recent work by Marx et al. [17] demonstrated that it was necessary to consider the eddy viscosity for the problem of acoustic modes traveling in a turbulent shear flow. By appropriately introducing the effects of eddy viscosity, the existence of an unstable mode was predicted, whose wavenumber and eigen-function profile agreed well with that of the experiment. Inspired by this, the eddy viscosity will be taken into account in this paper. The effect of viscosity was also considered in several numerical studies dealing with the acoustic response of a turbulent shear layer [18–20], and the similar kind of formalism will be used here.

In this paper, a numerical study of acoustic-flow interaction is carried out for the problem of sound propagation in a partly-lined duct with shear flow using a methodology based on frequency-domain linearized Navier–Stokes equations. Detailed description of the model is available in Section 2. The Galerkin finite element method is employed and the eddy viscosity is considered in the model. Section 3 gives the results and discussions. The parameters are taken to be the same as those in [13], so that the numerical results can be compared to the experimental ones. Particular attention is paid on the effect of the viscosity as well as the velocity profile on the hydrodynamic instability. Finally, conclusions are given in Section 4.

2. Model description

As shown in Fig. 1, the presence model considers a two-dimensional duct where the lined wall lies in the region of $y = H$ and $x_0 \le x \le x_L$ in the Cartesian coordinate system. The wall apart from the lined part of the duct is rigid. A subsonic mean flow is present in the duct along the x-direction with Mach number profiles $M_R(y)$ above the rigid wall and $M_L(y)$ above the lined wall, respectively.

2.1. Governing equations

In the present of turbulent shear flow, the effect of turbulence on the coherent perturbations (e.g. sound wave propagation) cannot be ignored ; hence, a triple decomposition is performed [21–23]:

$$R = \overline{R} + \tilde{R} + R' \quad (1)$$

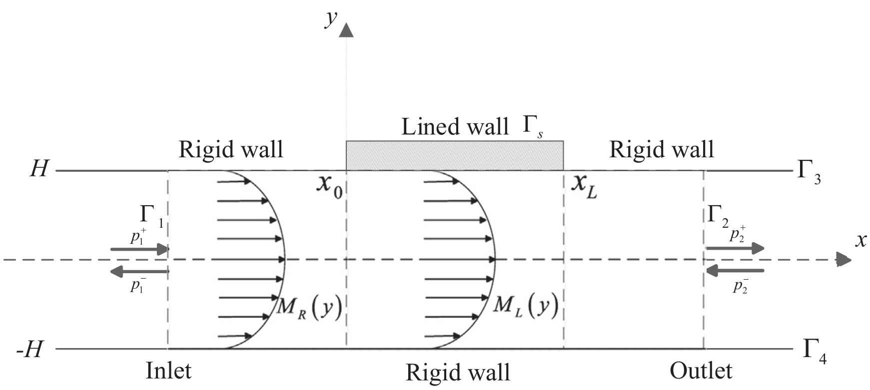

Fig.1. Sketch of the studied configuration.