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A systematic asymptotic approach to determine the dispersion characteristics of structural-acoustic waveguides with arbitrary fluid loading

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ABSTRACT

Structural-acoustic waveguides of two different geometries are considered: a 2-D rectangular and a circular cylindrical geometry. The objective is to obtain asymptotic expansions of the fluid–structure coupled wavenumbers. The required asymptotic parameters are derived in a systematic way, in contrast to the usual intuitive methods used in such problems. The systematic way involves analyzing the phase change of a wave incident on a single boundary of the waveguide. Then, the coupled wavenumber expansions are derived using these asymptotic parameters. The phase change is also used to qualitatively demarcate the dispersion diagram as dominantly structure-originated, fluid-originated or fully coupled. In contrast to intuitively obtained asymptotic parameters, this approach does not involve any restriction on the material and geometry of the structure. The derived closed-form solutions are compared with the numerical solutions and a good match is obtained.

1. Introduction

To understand the fluid–structure interaction in structural-acoustic systems, the analytical treatment of the dispersion relation is found to be insightful [1,2]. In this regard, asymptotic methods have been used in several studies [3–10] in order to solve the coupled dispersion relation and understand its dispersion characteristics. In asymptotic methods, a small (or large) parameter is required to carry out the calculation. And most often, researchers have found the small parameter either from physical intuition or by trial and error. The obtained expansions work well in certain frequencies and fail to do so at others. Of course, at the instances where the expansions fail, other expansions can be found. This again depends on intuition, experience and trial and error.

A typical example is by Sarkar et al. [9] where a fluid-loading parameter μ was introduced for a cylindrical structural-acoustic waveguide. μ is given by

$$\mu = \frac{\rho_F a}{\rho_S h}, \tag{1}$$

where ρ_F is the density of the fluid, ρ_S is the density of the structural material, h is the thickness of the shell and a is shell

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radius. Coupled wavenumber expressions were obtained for $\mu = 0.2$, i.e., $\mu \ll 1$ and $\mu = 5$, i.e., $\mu \gg 1$ for the $n=0$ mode using the Donnell–Mushtari (DM) theory. The analysis was done at low and high frequencies. The μ values and the frequency ranges considered being extreme were convenient for asymptotic analysis. For the same fluid-filled cylindrical shell with steel–water components, Fuller et al. [11] obtained dispersion characteristics numerically for $n=0$ and $n=1$. For a steel–water combination $\mu = 2.56 = \mathcal{O}(1)$ (for $h/a = 0.05$). It was found that the coupled wavenumbers were perturbations to either the uncoupled rigid-duct modes or uncoupled pressure-release wavenumbers or the *in vacuo* structural wavenumbers at certain frequencies. At a value $\mu = 2.56$, since perturbed solutions were still obtained, it is not clear whether $\mu = 2.56$ is small, large or neither in terms of being a uniform asymptotic parameter.

Further, the same system as the above was subjected to analytical treatment in Ref. [12] without any restriction on μ and curves were computed for the particular value $\mu = 2.56$ using perturbation methods. The wavenumber expressions were obtained either away from or in the neighborhood of the *in vacuo* wavenumbers. The curves were found to be perturbations to rigid-duct wavenumbers and the *in vacuo* structural wavenumbers. However, a few dispersion curves did not match with the numerical solutions (in Figs. 3(b) and 4(b) of Ref. [12]). It turns out that for the same value of $\mu = 2.56$, the curves follow the pressure-release curves for low frequencies and the rigid-duct curves for the high frequencies. Thus, the intuitively used parameter μ does not across the frequency range remain as an asymptotic parameter.

An alternative approach is to first categorize the wavenumber–frequency space as structure-originated, fluid-originated or fully coupled (i.e., dominated by both) based on physical arguments. Here it is not the value of a single intuitive parameter that is in question. Hence, there are no restrictions on physical parameters (such as densities or the material constants). This approach is applicable for a wide range of thin structures (for example, isotropic or orthotropic) and for an arbitrary μ. This qualitative approach is then translated into quantitative asymptotic parameters that can be used to derive wavenumber expansions.

In this paper, initially we consider a 2-D rectangular waveguide with a flexible upper boundary and a lower rigid boundary. For this system, the wavenumber–frequency space is demarcated into uncoupled, partially coupled or fully coupled regions by analyzing the phase change of a plane wave incident only on the flexible boundary. The other boundary is ignored. By doing this, the problem becomes simplified since the cross-sectional modes do not appear in the calculation. This approach is inspired by the work of Shu and Ginsberg [13]. They used a similar approach, again in a rigid rectangular waveguide to study the finite-amplitude wave propagation. This analysis results in identifying the appropriate asymptotic parameters at different frequencies in the wavenumber–frequency space. Following this, using these obtained asymptotic parameters, the coupled wavenumber expansions are derived. Next, a more complex system, a circular cylindrical waveguide is considered and the same analysis is conducted. Asymptotic wavenumber expansions are obtained by choosing appropriate asymptotic parameters. These parameters (obtained through a systematic procedure) are compared with those obtained using intuition in Ref. [12] and the corresponding results are discussed.

In Section 2, the 2-D rectangular waveguide is considered and the coupled dispersion relation is derived. Next, in Section 2.1, the effect of the structural boundary is studied by analyzing the phase change it causes on an incident plane wave. Based on the phase change, the dispersion diagram is categorized into various regions in Section 2.1.2. Next, in Section 2.1.3, the asymptotic parameters are obtained from the phase change in order to solve the coupled dispersion equation. Using these asymptotic parameters, the closed-form wavenumber solutions are obtained in Section 2.2. Similarly, in Section 3, an orthotropic circular cylindrical shell is considered. The dispersion diagram is categorized based on the phase change of an incident plane wave on the cylindrical structural boundary. Based on the phase change, the asymptotic parameters are identified and the coupled wavenumber expansions are obtained.

2. Two-dimensional rectangular waveguide

Let us consider a two-dimensional rectangular waveguide with a 1-D elastic plate (along $\hat{x}$) and a rigid surface as its boundaries [8], as shown in Fig. 1. Let the uniform width of the cross-section (along $\hat{z}$) be a. The acoustic potential $\hat{\phi}$ is given by,

$$\hat{\phi} = \hat{A}e^{i(-\hat{\zeta}\hat{z} + \hat{\kappa}\hat{x} - \hat{\omega}\hat{t})} + \hat{B}e^{i(\hat{\zeta}\hat{z} + \hat{\kappa}\hat{x} - \hat{\omega}\hat{t})}, \tag{2}$$

where $\hat{\kappa}$ is the coupled wavenumber in the $\hat{x}$ direction, $\hat{\zeta}$ is the corresponding wavenumber along $\hat{z}$ and $\hat{\omega}$ is the frequency. The structural equation at $\hat{z} = 0$ is given by

$$\left(\frac{Eh^3}{12(1-\nu^2)}\frac{\partial^4}{\partial \hat{x}^4} + \rho_S h \frac{\partial^2}{\partial \hat{t}^2}\right)\hat{w} = -\hat{p}|_{\hat{z}=1}, \tag{3}$$

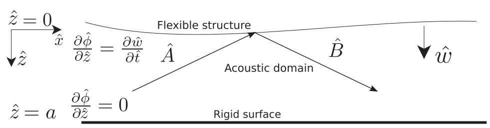

Fig. 1. A plane wave incident and reflected from the elastic boundary of a 2-D waveguide.