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On the dimension of complex responses in nonlinear structural vibrations

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ABSTRACT

The ability to accurately model engineering systems under extreme dynamic loads would prove a major breakthrough in many aspects of aerospace, mechanical, and civil engineering. Extreme loads frequently induce both nonlinearities and coupling which increase the complexity of the response and the computational cost of finite element models. Dimension reduction has recently gained traction and promises the ability to distill dynamic responses down to a minimal dimension without sacrificing accuracy. In this context, the dimensionality of a response is related to the number of modes needed in a reduced order model to accurately simulate the response. Thus, an important step is characterizing the dimensionality of complex nonlinear responses of structures.

In this work, the dimensionality of the nonlinear response of a post-buckled beam is investigated. Significant detail is dedicated to carefully introducing the experiment, the verification of a finite element model, and the dimensionality estimation algorithm as it is hoped that this system may help serve as a benchmark test case. It is shown that with minor modifications, the method of false nearest neighbors can quantitatively distinguish between the response dimension of various snap-through, non-snap-through, random, and deterministic loads.

The state-space dimension of the nonlinear system in question increased from 2–to–10 as the system response moved from simple, low-level harmonic to chaotic snap-through. Beyond the problem studied herein, the techniques developed will serve as a prescriptive guide in developing fast and accurate dimensionally reduced models of nonlinear systems, and eventually as a tool for adaptive dimension-reduction in numerical modeling. The results are especially relevant in the aerospace industry for the design of thin structures such as beams, panels, and shells, which are all capable of spatio-temporally complex dynamic responses that are difficult and computationally expensive to model.

1. Introduction

The maturation of finite element analysis (FEA) and the increase in computing speeds in the preceding decades have brought the concept of a ‘digital twin’ [1] closer to reality than ever before. Despite these advances, modeling coupled aero-thermo-structural response of high-speed aircraft using high-fidelity structural and computational fluid dynamics models is intractable, and will remain so for the foreseeable future [2]. Part of the problem lies in the fact that extreme loads induce

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nonlinear behavior, and thus superposition and scaling are no longer valid. In the case of hypersonic aircraft, wind-tunnel testing and prototype flights are also prohibitively expensive meaning that simulation results need to play a strong role in the design stage of potential aircraft. The single-discipline modeling of the structural behavior on its own remains a computational challenge. In [3], it is shown that high-fidelity finite element models (FEMs) can accurately reproduce complex nonlinear experimental behaviors such as snap-through. This work includes a comparison of the snap-through boundaries obtained experimentally and using FEA for a curved beam (representing buckled or curved panels), showing that FEA is in the position to do parametric studies, which is essential in design. The computations, however, still required several days, and the results were for only a single beam and under single-discipline modeling, as opposed coupled modeling of an acreage or preferably an entire aircraft.

Reduced order models (ROMs) show promise in reducing the computational costs of dynamical systems models. This approach attempts to extend modal analysis methods of linear dynamical systems by using a combination of either the linearized mode shapes, snapshots of either the static or dynamic nonlinear response, or a combination thereof to approximate the spatial configuration of a structure [4]. Reduced order models are usually calibrated using either experimental results or high-fidelity FEM results. Unfortunately, the possibility of bifurcations in the response type of a structure under a change in system parameters, or sensitivity to initial conditions and coexisting response types means that ROMs offer no guarantee of robustness. This has led to work in the area of adaptive ROMs. For example, [5] looks at creating adaptive thermal ROMs, where enrichment modes determined via proper orthogonal decomposition (POD) are added (and potentially subtracted) to the thermal model. This early work in the area of adaptive ROMs shows potential. The determination of a metric that indicates when a model needs enrichment (or when it may be simplified) is important for the development of adaptive schemes in vibration and snap-through problems (not done in [5]).

Whether for calibrating a ROM, or for developing an adaptive framework, it is important that the dimensionality of the structural response is well understood. The study of the dimensionality of time series is a mature field. Much of the literature in this field relies on statistical approaches such as POD (which is similar to Karhunen Loéve decomposition, principle component analysis, or singular value decomposition among many other terms) [6]. Statistical approaches work well in many applications in medical research [7], and have long been used in image processing [8]. These non-parametric statistical methods, however, do not have any connection to a dynamics model. An alternative, more physics-based approach, is the false nearest neighbors (FNN) method. The FNN method works by comparing whether the nearest neighbors of a phase portrait in state space at some dimension, d, are still nearest neighbors at dimension $d+1$. In the event that neighbors are no longer nearby in the higher dimension, it can be concluded that they only appeared to be near each other (in state space) because the reconstruction in dimension d was a projection. The dimension of the state space is continually increased until the number of false neighbors becomes low enough for the analysts intended purpose. The method works because the velocity field of a dynamical system in the completely unfolded state space must be unique, i.e. only in a projection of a phase portrait will two trajectories emanate from the same point. An excellent discussion with graphic illustrations may be found in [9]. Further discussion of the method with various metrics used to distinguish false from true neighbors may be found in [10–13]. The introduction of metrics and thresholds is a fundamental problem with the method of FNN, as there is often some subjectivity in distinguishing false from true neighbors in state space.

There are several important papers in the literature on chaos identification [14–16], which rely on FNN to reconstruct a phase portrait for eventual calculation of Lyapunov exponents. The potential of chaotic orbits to aid in developing ROMs is also discussed in [17]. The phase portraits of chaotic orbits are typically higher-dimensional than periodic or quasi-periodic responses, and thus exercise the dynamical system and provide mode shapes that better span phase space.

The method of FNN relies on time lag embedding to reconstruct the state space. Given a time series of an observable (in this context this is the displacement or velocity measured at a single point on a structure) of an n dimensional phase portrait, Takens' theorem [18,13] guarantees that time lag embedding will render a phase portrait that is topologically equivalent to the true state space if the embedding dimension is selected to be at least $2n$, although in practice lower dimensions may be sufficient. Thus, the reconstructed phase portrait will have the same dimension as the more natural position/velocity phase portraits.

The primary downside of the method of FNN is that it is computationally expensive, especially in higher-dimensional systems. This negative aspect is more than made up for by the fact that, unlike POD, the dimensionality of a response may be approximated using only a single observable, such as the displacement or velocity at the midspan of the beam. This makes it well-suited to applications where data is available for only a small set of points on a distributed structure. This is frequently the case, due to the high cost of digital image correlation (DIC) and scanning laser technologies, and constrained experimental configurations such as wind tunnel tests or prototype aircraft flights. According to the Mañé theorem [19], the dimension of the state space should be $D_p = 2D_c + 1$, where D_c is the number of independent active modal coordinates (which might be determined using POD). This is not hard to imagine, as in state space each modal coordinate also has a modal velocity, and one is added for time (in forced systems).

The metrics used to distinguish false from true nearest neighbors that are available in the literature work very well for low-order benchmark systems such as the Lorenz equations, but have difficulty when extended to higher-dimensional systems. This is because in high-dimensional space trajectories are less likely to pass very near to each other than in a smaller space. There is also always a possibility of corruption due to noise, i.e. noise may make true near neighbors appear far apart or vice versa [20]. Successful attempts at determining the embedding dimension in distributed (effectively high-dimensional in the FEM sense) response types were made in [21,22], showing that the embedding dimension agrees with the results of POD.