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Harmonic balance analysis of nonlinear tristable energy harvesters for performance enhancement

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ABSTRACT

Nonlinear energy harvesters are very sensitive to ambient vibrations. If the excitation level is too low, their large-amplitude oscillations for high-energy voltage output cannot be obtained. A nonlinear tristable energy harvester has been previously proposed to achieve more effective broadband energy harvesting for low-level excitations. However, the sensitivity of its dynamic characteristics to the system parameters remains uninvestigated. Therefore, this paper theoretically analyzes the influence of the external load, the external excitation, the internal system parameters and the equilibrium positions on the dynamic responses of nonlinear tristable energy harvesters by using the harmonic balance method. In addition, numerical acceleration excitation thresholds and basins of attraction are provided to investigate the potential for energy harvesting performance enhancement using the suitable equilibrium positions, appropriate initial conditions or external disturbances, due to high-energy interwell oscillations in the multi-solution ranges. More importantly, experimental voltage responses of a given tristable energy harvester versus the external excitation frequency and amplitude verify the existence of experimental multi-solution ranges and the effectiveness of the theoretical analysis. It is also revealed that achieving high-energy interwell oscillations in the multi-solution ranges of tristable energy harvesters will be feasible for improving energy harvesting from low-level ambient excitations.

1. Introduction

Piezoelectric vibration energy harvesting has been receiving more and more attention over the last decade [1,2]. This technology is considered to be a promising way to provide the energy supply for small sensors and MEMS devices [3,4]. A series of different linear resonance-based piezoelectric energy harvesters have been designed to generate electric energy by harvesting ambient vibrations. Reliable theoretical modeling methods for such harvesters have been proposed to predict their dynamic characteristics [5–8]. However, a challenging issue of such resonant harvesters is that they will not efficiently harvest ambient vibration energy when the excitation frequency shifts from their resonant frequency range. To solve this problem, many researchers have been focusing on widening the effective frequency range of energy harvesters via active or adaptive frequency-tuning schemes, as shown in the review papers [9–13]. An extensive investigation of broadband

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vibration energy harvesters with nonlinear monostable [14–20], bistable [21–32] and tristable [33–37] characteristics has being carried out in detail.

Due to high-energy interwell oscillations, the efficient energy harvesting performance of bistable energy harvesters can be greatly enhanced [9–13]. McInnes [21] developed a theoretical model of bistable harvesters using stochastic resonance mechanism to improve the energy harvesting performance. The research provided by Cottone et al. [22] demonstrated that nonlinear bistable energy harvesters may have advantages over their linear counterparts when subject to stochastic excitations. Daqaq [23] theoretically investigated the response of bistable energy harvesters under white and exponentially correlated Gaussian noise excitations, and their research concluded that the potential shape had little influence on the expected power. Cao et al. [24] used frequency analysis, bifurcation diagrams and Poincare maps to analyze the chaotic and periodic dynamic responses of bistable harvesters under harmonic excitations. Meanwhile, Litak et al. [25] numerically investigated the effect of the mechanical damping for bistable systems under stochastic noise excitations. Kwuimy et al. [26] presented a nonlinear analysis of a bistable energy harvester whose stiffness and inductance are of fractional-order. Panyam et al. [27] and Stanton et al. [28,29] theoretically and numerically analyzed the nonlinear dynamic characteristics of bistable energy harvesters. Friswell et al. [30] and Borowiec et al. [31] studied a nonlinear elastic inverted pendulum including a vertically oriented beam, which could be buckled under the gravitational load of a proof mass attached to its end. Their results demonstrated that such system would exhibit bistable characteristics when the proof mass is appropriate. Erturk et al. [32] investigated magnetically coupled bistable energy harvesters whose high-energy interwell oscillations could effectively enhance vibration energy harvesting. However, bistable energy harvesters cannot perform high-energy interwell oscillations when subject to low level excitations [12,13].

In order to enhance energy harvesting performance under low-level excitations, Zhou et al. [33] designed a tristable energy harvester with shallower potential wells, and their results demonstrated the better and more practical energy harvesting results of the tristable energy harvester when compared with its bistable counterpart. The investigation into the influence of potential well depth on tristable energy harvesting performance is numerically and experimentally provided by Cao et al. [34]. Kim et al. [35] numerically showed their tristable oscillator constructed around a cantilever-based magnetically coupled system, and their simulations also demonstrated that the advantages of tristable energy harvesters. Tékam et al. [36] discussed a tristable energy harvesting system with viscoelastic material modeled by using fractional derivatives, and investigated the influence of the fractional-order term. However, both bistable and tristable energy harvesters cannot output sufficient voltages when subject to very low-level excitations because of the constraint of their potential barriers. In order to overcome this issue, Zhou et al. [37] proposed an initial impact method to cause bistable and tristable energy harvesters to realize high-energy interwell oscillations under very low-level excitations. Nevertheless, there are several remaining issues regarding the effect of parameter variations on the dynamic responses of tristable energy harvesters. Understanding these effects will help in optimizing tristable energy harvesters in various applications. Therefore, further theoretically investigation of the fundamental nonlinear dynamic characteristics and the influence of parameter variations of tristable energy harvesters is warranted and presented here.

The paper is organized as follows. The next section provides the derivation of the harmonic balance solutions and the expression of the Jacobian matrix that determines the stability of the harmonic balance solutions to theoretically analyze the

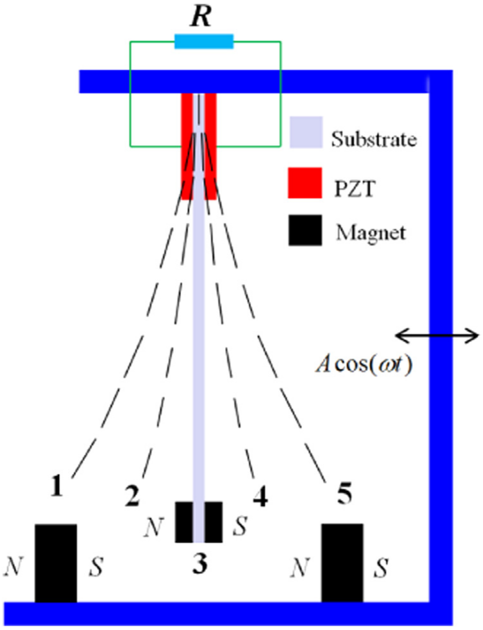

Fig. 1. Schematic of the tristable energy harvester.