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Characterization of acoustic black hole effect using a one-dimensional fully-coupled and wavelet-decomposed semi-analytical model

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ABSTRACT

Acoustics Black Hole (ABH) effect shows promising features for potential vibration control and energy harvesting applications. The phenomenon occurs in a structure with diminishing thickness which gradually reduces the phase velocity of flexural waves. The coupling between the tailored ABH structure and the damping layer used to compensate for the adverse effect of the unavoidable truncation is critical and has not been well apprehended by the existing models. This paper presents a semi-analytical model to analyze an Euler-Bernoulli beam with embedded ABH feature and its full coupling with the damping layers coated over its surface. By decomposing the transverse displacement field of the beam over the basis of a set of Mexican hat wavelets, the extremalization of the Hamiltonian via Lagrange's equation yields a set of linear equations, which can be solved for structural responses. Highly consistent with the FEM and experimental results, numerical simulations demonstrate that the proposed wavelet-based model is particularly suitable to characterize the ABH-induced drastic wavelength fluctuation phenomenon. The ABH feature as well as the effect of the wedge truncation and that of the damping layers on the vibration response of the beam is analyzed. It is shown that the mass of the damping layers needs particular attention when their thickness is comparable to that of the ABH wedge around the tip area. Due to its modular and energy-based feature, the proposed framework offers a general platform allowing embodiment of other control or energy harvesting elements into the model to guide ABH structural design for various applications.

1. Introduction

Developing highly-damped and light-weighted structures is of great importance for various engineering problems. Traditional methods such as viscoelastic coating for structural damping enhancement usually require covering structural surface over a large area, thus leading to additional weight [1]. The approach using a graded impedance interface for attenuating structural wave reflections at the edges of plates and bar [2] tackles the abovementioned drawback of traditional methods, but is restricted in practical application due to the technical difficulties in creating suitable impedance

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interfaces. Mironov [3] first proposed the concept of Acoustics Black Hole (ABH) by reducing structure thickness according to a power-law profile (with power no less than 2) to gradually reduce the local phase velocity of the flexural waves, achieving zero reflection in the ideal scenario thus creating energy concentration at the tip end [3–5]. ABH effect shows appealing features in vibration control because only a very small amount of damping materials is required at the energy focalization region to achieve efficient damping of flexural waves [6–10]. In addition, it also shows potential in sound radiation control [11,12] and energy harvesting due to the high energy concentration within a confined area [13]. Periodic ABH profiles can also be included in structures to further increase the overall performance [10]. In these applications, on one hand, the addition of vibration control or energy harvesting elements may affect the formation of the ideal ABH through their interference with the host structure; on the other hand, topological or system optimizations may be needed to achieve the maximum performance. To this end, a flexible model, which allows the consideration of the full coupling between the host ABH structure and various control or energy-harvesting elements to be embedded, is of paramount importance.

Among existing models describing the ABH effect, the geometrical acoustic approach [14] was first proposed to analyse the flexural wave propagation in tailored wedges and to calculate the reflection coefficients [6,7] on the premise of smoothness assumption of the thickness profile [15]. The wave would never be reflected back if the thickness of wedge was ideally reduced to zero. However, the unavoidable truncations in real fabricated wedges would significantly increase the reflection coefficients, which can be compensated to a certain extent by covering the wedge surfaces with thin damping layers [6,7]. An impedance method which is not limited by the hypothesis of geometrical acoustics has also been proposed by Georgiev et al. for beam structures, which ultimately leads to the reflection matrix by Riccati equation [16,17]. These two types of approaches only consider semi-infinite structures, even only the ABH wedge part in some cases [6,7]. This is obviously different from the practical situation in which structures are finite in size with real boundary, and an ABH profile is usually only part of conventional structures. All these combined, multiple reflections take place between boundaries as well at the intersection between the ABH portion and the rest of the structure, which cannot be apprehended by the existing models. On the other hand, existing approaches consider the effect of a thin damping layer through Ross–Unar–Kerwin (RUK) model [1], which assumes the thickness of the damping layer is much smaller than that of the wedge. In practice, however, the thickness of even an extremely thin damping layer would be comparable to that of the wedge tip, where ABH effect is the largest, which suggests the importance of considering a few practical issues. The first one is the possible increasing importance of the added mass effect, which has been considered in some previous work [7] for a uniform layer. If one wants to further optimize the damping layer through adjusting its location and shape to achieve maximum damping performance, the added mass and stiffness effect of the damping layer needs to be treated in a more versatile manner. Most importantly, the unavoidable full coupling between the damping layer and the power-law profile wedge might need to be considered to better reflect the reality. This issue becomes even more important when other control and energy harvesting elements are added. The geometrical and material characteristics as well as the location of damping layers are shown to greatly affect the performance of damping layers on energy dissipation [17,18]. An optimization on these parameters as well as the thickness variation of the damping layers might be an additional way to achieve the maximum energy dissipation.

In summary, the full coupling between the damping layers and the ABH taper needs to be considered. Meanwhile, the consideration of more realistic structures with finite size and boundary is necessary to guide the design of practical ABH structures. To this end, a simulation model is necessary, which can truthfully characterize the ABH phenomenon while offering the flexibility of considering additional control and energy harvesting elements for further potential applications.

In this paper, we propose a semi-analytical model to analyze an Euler–Bernoulli beam containing a portion with embedded ABH feature and its full coupling with a thin damping layer over its surface. The beam is of finite length with arbitrary boundary conditions. The speed of the flexural waves and the wavelength remain constant in the uniform portion of the beam. When entering into the tapered region, however, the thickness reduction of the beam reduces the wave speed rapidly, along with a much shortened wavelength. This non-uniform and fast-varying nature of the wavelength creates particular challenges to the modeling. To tackle the problem, a wavelet-decomposed formulation is proposed in this paper. This model takes the damping layer as an integral part of the system, thus conserving its full coupling with the host structure. Meanwhile, due to its energy-based and modular nature, it allows easy extension to further include other embedded control or energy harvesting elements for potential applications. Via Lagrange's equation, Mexican hat wavelets are proposed to decompose the displacement field of the system, leading to the theoretical model presented in Section 2. In Section 3, numerical results are compared with FEM for validation. The ABH features, the effects of the truncation, damping layer and full coupling are investigated. Meanwhile, a preliminary analysis on the location and the shape of the damping layers is carried out to illustrate the versatility of the model. The numerical results from present model are further compared with experimental measurements to confirm the accuracy of this model in Section 4. Finally, conclusions are drawn in Section 5.

2. Theoretical model and formulation

2.1. Modeling procedure

As shown in Fig. 1, consider an Euler–Bernoulli beam undergoing flexural vibration under a point force excitation $f(t)$ at x_f. The response is measured at point x_m. The beam is composed of a uniform portion with constant thickness h_b from x_{b1} to