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Period of vibration of axially vibrating truly nonlinear rod

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ABSTRACT

In this paper the axial vibration of a muscle whose fibers are parallel to the direction of muscle compression is investigated. The model is a clamped-free rod with a strongly nonlinear elastic property. Axial vibration is described by a nonlinear partial differential equation. A solution of the equation is constructed for special initial conditions by using the method of separation of variables. The partial differential equation is separated into two uncoupled strongly nonlinear second order differential equations. Both equations, with displacement function and with time function are exactly determined. Exact solutions are given in the form of inverse incomplete and inverse complete Beta function. Using boundary and initial conditions, the frequency of vibration is obtained. It has to be mentioned that the determined frequency represents the exact analytic description for the axially vibrating truly nonlinear clamped-free rod. The procedure suggested in this paper is applied for calculation of the frequency of the longissimus dorsi muscle of a cow. The influence of elasticity order and elasticity coefficient on the frequency property is tested.

1. Introduction

Nowadays, a significant number of investigations into the elasticity of muscles are carried out, as it is believed that tissue elasticity has direct relevance in the early detection of diseases [1,2]. Namely, there are numerous diseases which significantly alter tissue elasticity, for example cancer. This is the reason that various methods for measuring for tendon, heart, skin, cartilage, liver, fat and muscle are developed [3–5]. Specially, elasticity is experimentally obtained for those muscles whose fibers are parallel to the direction of muscle stretch or compression [6], i.e., deformation orientation is axial. In Fig. 1, the stress–strain diagram for an axially loaded muscle sample excised from the beef longissumus dorsi muscle is plotted. (The longissimus is the longest subdivision of the erector spinae which in animals and humans extends the vertebral column.)

Dimensions of the sample were 4 cmx4 cm in length and width, with thicknesses ranging from 1 to 2 cm. The elasticity diagram is experimentally obtained by a standard Instron load cell tensile testing machine. It can be seen that the elasticity function is strongly nonlinear (Fig. 1b) and the muscle has to be treated as a system with strongly nonlinear property. It has to be addressed that the stress–strain diagram given in [6] is for compression and for this type of biological tissue the strains are only with one sign [7]. Comparing the experimental result with that obtained with established mechanical procedure it is concluded that the accuracy of the first is not enough high. The maximal relative error is up to 75%. This may cause

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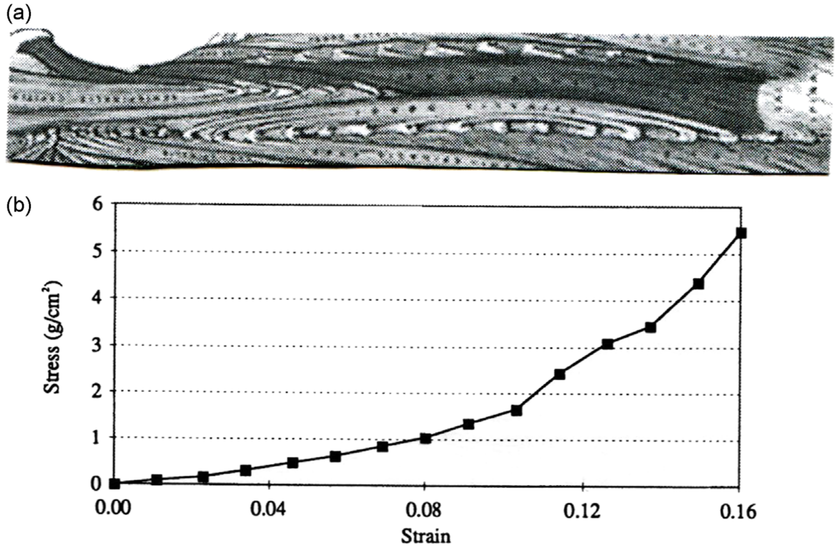

Fig.1. (a) Longissimus muscle (black) and (b) stress–strain diagram [6].

troubles in the clinical diagnostics of diseases. To eliminate this disadvantage, it is suggested to monitor the vibration property (frequency or period) of the muscle, too. This value is in correlation with the elasticity of tissue and may be a useful data for decision about illness or health.

A muscle with parallel fibers has to be treated as an axially vibrating rod with nonlinear elastic properties. Namely, the rod is assumed to be a clamped-free one. Free vibrations of the rod are analyzed. Then, the effect of nonlinearity and the elastic properties on the frequency of vibration is obtained.

Axial vibration of the nonlinear rod has already been investigated by the author [8], but the nonlinearity was of a different type. Unfortunately, the developed procedure for solving such nonlinear equations is not applicable in general for all types of nonlinearity.

This paper is organized in seven sections. After the introduction, the axial vibrating rod is modeled and axial vibration is mathematically described (Section 2). The model is a partial second order strongly nonlinear differential equation with two linear boundary and two initial conditions. In Section 3, a solution procedure for this partial differential equation is developed. The vibration function is assumed as a product of a displacement and of a time function. So, the partial differential equation of motion is separated into two independent strongly nonlinear ordinary second-order differential equations. Solution of the equation with displacement function is obtained in the form of the inverse incomplete Beta function, while of the equation with time function in the form of the Ateb-function. Based on initial conditions and solutions of both equations (with displacement and with time), the constants of integration are calculated. They affect the frequency of axial vibration of the clamped-free rod (Section 4). In this section we investigate the variation of the vibration frequency due to change of the frequency constant and elasticity properties of the system. In Section 5, a numerical simulation of the axial vibration is shown. Using the *Mathematica* software, the solution of the equation, even with high order of nonlinearity (3 and higher), is plotted. The influence of the order of nonlinearity is discussed. Special attention is given to the frequency of vibration (see Section 6) of the longissimus muscle of a cow. We discuss the variation of the frequency of muscle caused by elasticity (variation of the order of nonlinearity) and coefficient of nonlinearity change which occur due to alterations in the tissue.

2. Model of the muscle

The muscle is modeled as a clamped-free rod (Fig. 2). Length of the rod is$l$ and its cross-section S. Cross-section properties of the rod are smaller than its length. Rod has axial vibrations. Deflection u of the rod depends on time t and position x. Elementary part of the rod has length dx and mass $\rho A dx$, where ρ is density of material. Inertial force of the elementary part is the product of elementary mass and acceleration: $\rho A dx(\partial^2 u/\partial t^2)$. Material of rod has strongly nonlinear properties and its stress–strain rheological model is given as

$$\sigma = E\varepsilon|\varepsilon|^{\alpha-1} = E\frac{\partial u}{\partial x}\left|\frac{\partial u}{\partial x}\right|^{\alpha-1}, \tag{1}$$

where E is the elasticity coefficient, ε is the deformation, $|\partial u/\partial x|$ is positive strain and $\alpha\epsilon[1,\infty)$ is the order of nonlinearity obtained experimentally (see Fig. 1b) which may be integer or non-integer.