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Computing the modal mass from the state space model in combined experimental–operational modal analysis

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ABSTRACT

Modal parameters comprise natural frequencies, damping ratios, modal vectors and modal masses. In a theoretic framework, these parameters are the basis for the solution of vibration problems using the theory of modal superposition. In practice, they can be computed from input–output vibration data: the usual procedure is to estimate a mathematical model from the data and then to compute the modal parameters from the estimated model. The most popular models for input–output data are based on the frequency response function, but in recent years the state space model in the time domain has become popular among researchers and practitioners of modal analysis with experimental data. In this work, the equations to compute the modal parameters from the state space model when input and output data are available (like in combined experimental–operational modal analysis) are derived in detail using invariants of the state space model: the equations needed to compute natural frequencies, damping ratios and modal vectors are well known in the operational modal analysis framework, but the equation needed to compute the modal masses has not generated much interest in technical literature. These equations are applied to both a numerical simulation and an experimental study in the last part of the work.

1. Introduction

The essential idea of modal analysis is to describe the vibrations of a structural system with simple components, the so-called vibration modes. Such is the importance of vibration modes that we could even say that understanding vibration problems requires understanding the modal properties.

Modal parameters are used by engineers in many applications, for example:

- *Model updating*: The experimental data recorded at a structural system is used to calibrate a mathematical model in order to obtain more reliably predictions of its dynamical behaviour [1].
- *Structural health monitoring*: Changes in the vibration pattern are indicative of changes in the structure, for example, due to damage [2,3].
- *Vibration serviceability and control of vibrations*: The vibration level of structural systems is measured and predicted for other scenarios. When comfort values are exceeded, specific devices are designed and incorporated to the system in order to reduce the vibrations [4,5].

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- *Load estimation*: Vibration measurements are used to estimate the forces acting in the structures [6].

In practice, the modal properties can be estimated from experimental data generally using measured displacement, velocities or accelerations [7]. Technical literature differentiates three different approaches [8]:

- *Experimental modal analysis* (*EMA*): The structure is excited by one or several measured dynamic forces, the response of the structure to these forces is recorded, and the modal parameters are extracted from the input and output measurements. Therefore, EMA methods are suitable for laboratory conditions, where the inputs, the outputs and the boundary conditions of the test are totally controlled and measured. In fact, EMA is a well-established and often used approach in mechanical engineering, as documented in [7,9,10].
- *Operational modal analysis* (*OMA*). In large structures, like bridges or buildings, it is difficult to apply known inputs. This led to the idea of using the vibrations due to environmental loads such as wind and traffic. Here the assumption that the unmeasured loads are realisations of a stationary stochastic process substitutes for the deterministic knowledge of driving forces. In technical literature, we can find different names for this technique:
  - *Ambient modal analysis*: the inputs acting in the structure are ambient inputs (wind, traffic, earthquakes, etc.), not controlled inputs (hammers, shakers, etc.).
  - *Output-only modal analysis*: the modal parameters are estimated using only the response of the structure.
  - *Operational modal analysis or in-operation modal analysis*: the vibrations are recorded when the structure is in its normal operation. This is the most used name in the literature and this is the name we are going to use in this work. In civil engineering, OMA has become the primary modal testing method, and the number of reported case studies is abundant: roadway and railway bridges [3,11–14], footbridges [15,16], buildings and special structures [17], historical buildings [18], offshore platforms [19], high-rise buildings [20], dams [21], among others.

- Combined experimental–operational modal analysis or Operational Modal Analysis with eXogenous inputs (OMAX). Last years, some researchers have used practical actuators in the modal test of large structures [16,22,23]. The amplitude of the artificial (measured) forces can be equal or even lower than the amplitude of the operational forces, and both operational and artificial forces are included in the identified system model.

Independent of the approach, performing modal analysis with experimental data typically consists of three basic steps [24]:

1. *Data collection*: sensors (and actuators depending on the test) are placed in the system and time series of vibration data are collected.
2. *System identification*: a mathematical model is fitted to the measured data. We need to distinguish between models and methods to estimate such models.
   - *Models*: In the frequency domain, the main model is the frequency response function (see [10] for an extensive description of this model and methods associated to it). In the time domain, we can find ARMAX models [25], state space models [26], etc.
   - *Estimation methods*: Popular estimation methods are Prediction Error Method (PEM), Least-Squares (LS) and Maximum Likelihood Estimation (MLE). For the state space model, examples of PEM can be found in [27], examples of LE algorithms are the Subspace algorithms [28], and the Expectation-Maximisation algorithm is an example for MLE [29].
3. *Extracting a set of modal parameters*: The modal parameters are computed from the estimated model using the equations that relate model and modal parameters.

This work focusses on the third item (computing the modal parameters from the estimated model) when two conditions are verified: first, input and output vibration data are available (like in combined experimental–operational modal analysis); second, the model estimated from input–output data is the state space model. Under these conditions, the equations are derived using invariants of the state space model.

The equations corresponding to natural frequencies, damping ratios and modal vectors match the ones used in OMA (see [26] for the derivation of these equations in an OMA framework): in fact, OMA can be considered as a particular case of OMAX. However, the knowledge of input data allows us to estimate the modal masses. The modal mass is an important parameter in modal analysis but its estimation has not generated much interests compared to natural frequencies, damping ratios and modal vectors. In classical experimental modal analysis, modal masses have been computed using frequency response functions [9]. These equations have been also used in recent combined experimental–operational modal tests [8,16,23]: they used the state space model to compute natural frequencies, damping ratios and modal vectors, and the theory of frequency response functions to compute modal masses. One example of computing the modal masses from the state space model can be found in [30], where the modal mass corresponding to the first mode was estimated in structures equipped with tuned mass dampers using H-infinity optimal model reduction. In operational modal analysis, where the inputs are unknown, the modal masses are estimated using auxiliary procedures: adding external masses to the structure [31], measuring the human load in laboratory and then applying this input in situ [32], or designing special devices [33].