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Influence of the spatial correlation structure of an elastic random medium on its scattering properties ☆

Shahram Khazaie [a,b], Régis Cottereau [a,*], Didier Clouteau [a]

[a] MSSMat, CNRS, CentraleSupélec, Université Paris-Saclay, 92290 Châtenay-Malabry, France
[b] Laboratoire M2P2 UMR 7340, Aix-Marseille Université, CNRS, École Centrale Marseille, France

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ABSTRACT

In the weakly heterogeneous regime of elastic wave propagation through a random medium, transport and diffusion models for the energy densities can be set up. In the isotropic case, the scattering cross sections are explicitly known as a function of the wavenumber and the correlations of the Lamé parameters and density. In this paper, we discuss the precise influence of the correlation structure on the scattering cross sections, mean free paths and diffusion parameter, and separate that influence from that of the correlation length and variance. We also analyze the convergence rates towards the low- and high-frequency ranges. For all analyses, we consider five different correlation structures that allow us to explore a wide range of behaviors. We identify that the controlling factors for the low-frequency behavior are the value of the Power Spectral Density Function (PSDF) and its first non-vanishing derivative at the origin. In the high frequency range, the controlling factor is the third moment of the PSDF (which may be unbounded).
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1. Introduction

Exploration of the structure of the Earth using seismograms recorded at the surface is a classical problem in geophysics. Its applications range from academic understanding of the interior of the Earth to more industrially-oriented questions related to oil exploration or CO_2 and nuclear waste sequestration. The seismograms typically start with a ballistic or coherent part corresponding to the first arrivals of P, S or Rayleigh waves. This coherent signal is then followed by an incoherent wave train called the Coda [20,2], with less definite and more randomized features. The analysis of short-period seismograms shows the predominance of multiply scattered waves in this part of the seismograms [3]. At long lapse times, a diffusion regime steps in, which is characterized by the equipartition of energies [4]. The time decay of the late arriving waves is then shown to be a characteristic of the medium, independent of the source and path effects as well as site conditions [5], which makes the Coda a good candidate for identification procedures. Since the original studies on the origin of the Coda [1,5], the spatial variation of the medium properties has been taken into consideration and stochastic modeling has been used as a tool for understanding the complexity of the wave fields in geophysical applications. Recently, there has been an outburst of papers numerically simulating wave propagation in random geophysical media [6–10] to complement the more classical theoretical considerations [11–14,3] for the study of the Coda. Similar trends are observed in other fields

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* Corresponding author.
E-mail address: regis.cottereau@centralesupelec.fr (R. Cottereau).

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of physics and engineering, where the decaying part of the signal is analyzed in terms of diffusion equation and used to gather information on the material. This is particularly true for analyses of the propagation of ultrasound in polycrystalline materials [15,16], room acoustics [17–20] and high-frequency vibrations of complex structures [21]. Other examples include non-destructive testing of concrete samples [22] and wave propagation in granular materials [23–25].

To model wave propagation in random media, three typical length scales are of interest: (i) the dominant wavelength λ; (ii) the typical size of the heterogeneities (correlation length ℓ_c for instance); and (iii) the propagation distance L. The parameters that are used to separate between the different scattering regimes are ℓ_c/λ and L/λ [14]. At least three important regimes can be identified: (1) the effective medium or homogenization regime in which $\ell_c/\lambda \ll 1$ and $L/\lambda \sim 1$; (2) the weakly heterogeneous or stochastic scattering regime in which $\ell_c/\lambda \sim 1$ and $L/\lambda \gg 1$ (with limited strength of the heterogeneities); and (3) the strongly heterogeneous white-noise regime in which $\ell_c/\lambda \ll 1$ and $L/\lambda \gg 1$. In the homogenization regime, a deterministic effective wave equation describes the wave propagation, where the effective stiffness is obtained as in classical stochastic homogenization in statics and the effective density is obtained by arithmetic averaging [26,27]. In the strongly heterogeneous regime, the impact of the strong heterogeneities is limited by the disparity in typical lengths. However, an asymptotic regime arises over long enough distances [14] which is mainly influenced by the variance and correlation lengths of the mechanical parameters of the wave equation. Finally, in the weakly heterogeneous regime, the similarity between the wavelength and the heterogeneity length ensures an efficient interaction between the wave and the medium but the strength of the fluctuations is limited so the waves do not become localized [28,13,29]. We concentrate in this paper on this weakly heterogeneous regime, which is influenced by the full power spectrum of the mechanical parameters, rather than only on the correlation length and variance.

The theory of radiative transfer of elastic waves is an approach to study the multiple scattering of waves in this regime [11,15,30,31,16,32]. The radiative transfer equations (RTE) describe the spatio-temporal evolution of the wave vector-dependent energy densities of the waves. Vector transport equations were firstly developed for polarized light waves in statistically isotropic media by Chandrasekhar [11]. They were later developed for elastic waves in isotropic media independently by Weaver [15] and Ryzhik et al. [32] with different approaches. In these equations, the scattering effects resulting from the randomness of the medium are described by scattering cross-sections, that are explicitly related to the power spectrum of the random mechanical parameters of the wave equation (density and Lamé parameters). Such power spectra are parameterized in general by a variance and a correlation length. However, these two numbers are not sufficient to describe completely a correlation model.

The objective of this paper is to discuss the influence of the correlation structure (that is to say the correlation normalized by the correlation length and variance) on the scattering cross sections. In particular, we want to quantify how this influence compares to that of the variance and correlation length. This work was partially done by various authors in the scalar case [13,3] but we wish to address the elastic case, for which scattering cross sections are much more complicated (see Section 2.5). In particular the existence of P and S modes for wave propagation in the background medium means that several cross-sections must be defined rather than just one. Understanding of the precise influence of the correlation structure is important for at least two kinds of applications: (i) for parameterization of the random medium in the context of an inverse problem and (ii) for design and creation of new meta-materials. When performing an inverse problem in a random medium, it is a priori necessary to identify the complete correlation structure. However, if the analysis shows that in the particular range of frequencies considered only a finite number of parameters of that correlation structure are significant, the complexity of the inverse problem can be drastically reduced. In the case of design of meta-materials, which comes down to an optimization problem, it is again crucial to have a good parameterization of the material to limit the complexity of the problem. Previous authors have tried to tackle this problem by considering simplifying assumptions [33] or one specific correlation model [34,3], but we concentrate here on comparison between several very classical correlation structures.

The outline of the paper is the following. In Section 2, we introduce the general wave equation and RTE, along with the scattering cross-sections. We also introduce five classical correlation structures (exponential, power-law, Gaussian, triangular and low-pass white noise) that will be compared throughout the paper. We will see that this set of correlation structures shows a wide range of behavior for the scattering cross sections. In Section 3, we study the influence of the power spectrum structure on the scattering cross-sections and perform the asymptotic analyses. Section 4 introduces the diffusion approximation and proposes similar analyses for the diffusion parameters.

2. Elastic wave propagation in isotropic random media

We consider the propagation of elastic waves in a medium with continuous isotropic random heterogeneities and locally isotropic constitutive behavior [35,12–14,3]. The consideration of discrete random media is possible by considering the appropriate correlation structures, but will not be discussed here (see for instance [36–39]). Likewise, the anisotropic case is fully treated in Baydoun et al. [40] but will be left aside here.

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