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On wave reflections and energetics for a semi-infinite traveling string with a nonclassical boundary support

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ABSTRACT

The free, linear, transverse vibrations of a semi-infinite axially moving string are considered in order to study the reflection of an incident wave and the damping properties of different types of boundary conditions such as free, fixed, spring–dashpot, and mass–spring–dashpot. To obtain the response to the initial conditions, the method of d'Alembert is applied. Furthermore, analytical expressions for the time-rate of change of the total mechanical energy of the system are derived. The obtained results give insight into the most efficient way of placing a boundary support depending on the direction of the transport velocity. Moreover, for nonclassical boundary conditions, the dynamics of the string is described by the relative values of the system parameters.

1. Introduction

In the present study, a uniform semi-infinite string with density ρ, which is pulled under constant speed V and tension P over a smooth support, is considered (see the schematic diagrams in Table 1). In real engineering slender systems, the length of the string is finite, where the reflections of waves occur at both boundaries. The interaction among the reflected waves does not allow in a simple way to study the effectiveness of a boundary support as a standalone device, while the semi-infinite span of the string does. Each time a wave interacts with the boundary, then this simple model for a semi-infinite string can be used to determine how much energy for that wave is dissipated at that boundary. More precisely, by looking at the reflected wave profile and the energetics of the model under consideration, one can make a conclusion about the efficiency of a placed boundary support as a vibration suppressor.

There are a plenty of examples studying the reflection phenomena in stationary strings with classical boundary conditions by the classical d'Alembert solution in the literature (see, for instance, [1,2]). Recently, Akkaya and van Horssen [3] employed this method to obtain the exact solutions for the semi-infinite stationary string with nonclassical boundary conditions. Additionally, the authors analyzed reflection and damping properties of the considered boundary conditions. Apart from these, a lot of research has been done on the dynamic analysis of axially moving strings (see, e.g., the classical and well-known papers [4–13]). For example, Wickert and Mote [10] derived a classical vibration theory for traveling string and beam models providing the exact expressions for their responses in closed form. Darmawijoyo and van Horssen [12]

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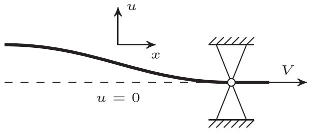
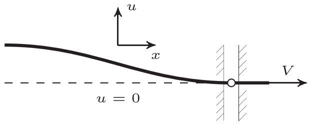
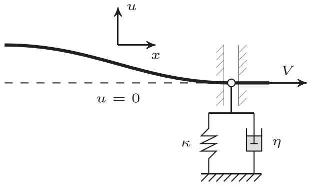
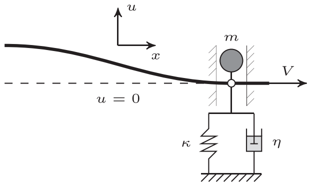
Table 1
List of various boundary conditions. The symbols are defined in the text.

Type	Diagram	Equation
Fixed		$u(0,t)=0$
Free		$u_x(0,t)=0$
Spring–dashpot		$\kappa u(0,t)+\eta u_t(0,t)+Pu_x(0,t)=\rho V[u_t(0,t)+Vu_x(0,t)]$
Mass–spring–dashpot		$mu_{tt}(0,t)+\kappa u(0,t)+\eta u_t(0,t)=\rho V[u_t(0,t)+Vu_x(0,t)]-Pu_x(0,t)$

constructed asymptotic approximations of the solution for an axially moving string with an attached spring–mass–dashpot system at its end. Chen [13] reviewed research over the last few decades on transverse vibrations of traveling strings and their control. Moreover, the author suggested future research direction in analysis and control of axially moving strings. Swope and Ames [14] derived a response for a moving threadline by the methods of d'Alembert and characteristics showing the importance of the relation between the winding speed and the wave velocity. Tan and Ying [15] used the transfer function formulation and the wave propagation concept to derive the exact response solution for the translating string with general boundary conditions. In the case of a dashpot boundary support, they observed complete wave absorption when the value of damping coefficient equals the propagation speed of the reflected wave. Lee and Mote [16] analyzed the energetics of translating continua for fixed, free, and damped boundary conditions. One of the main results is the evaluation of the energy by the reflection coefficients which are defined by the boundary conditions. Chen and Ferguson [17] studied the energy dissipation by a viscous damper attached at one end of a moving string. The authors could obtain the optimum value of this damper which dissipates most input energy for a constant as well as for a varying length of the string.

In contrast to previous research for traveling strings defined on a finite domain, our work focuses on reflection and damping properties of a single boundary of different types. Therefore, we choose a semi-infinite string as a model for consideration. This paper is organized as follows. Section 2 introduces the equations of motion describing the transverse vibrations of an axially moving semi-infinite string and different types of boundary conditions. In Section 3, we use the method of d'Alembert to obtain the response to the initial conditions. Additionally, we analyze the reflections of waves at different types of boundary supports. Next, Section 4 presents the total energy and its time-rate of change providing more insight into the stability of the system. Finally, Section 5 emphasizes the advantages of the used method for solution and summarizes the results obtained in Sections 3 and 4.

2. Equations of motion

The transverse equation of motion of the semi-infinite string can be obtained by the application of *Hamilton's principle* (see, for instance, [4,18,19]):

$$u_{tt}+2Vu_{xt}+(V^2-c^2)u_{xx}=0, \tag{1}$$

where u is the transverse displacement and $c=\sqrt{P/\rho}$ is the wave propagation speed. To avoid the divergence instability [16] in the string and to have a reflection of the incident wave at the boundary, we assume that the transport speed V is less than the critical one, i.e., $|V|<c$. To put the governing equation in a non-dimensional form, we incorporate the following