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A fully consistent linearized model for vibration analysis of rotating beams in the framework of geometrically exact theory

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ABSTRACT

The equations of motions governing the free vibrations of prismatic slender beams rotating in a plane at constant angular velocity are derived according to a geometrically exact approach. Compared to other modeling methods, additional stiffening terms induced by pre-stress are found in the dynamic equations after fully consistent linearization about the deformed equilibrium configuration. These terms include axial, bending and torsional stiffening effects which arise when second-order generalized strains are retained. It is shown that their contribution becomes relevant at moderate to high angular speeds, where high means that the equilibrium state is subject to strains close to the limit where a physically linear constitutive law still applies. In particular, the importance of the axial stiffening is specifically investigated. The natural frequencies as a function of the angular velocity and other system parameters are computed and compared with benchmark cases available in the literature. Finally, the error on the modal characteristics of the rotating beam is evaluated when the linearization is carried out about the undeformed configuration.

1. Introduction

The modeling of elastic bodies undergoing large rotational motion has been widely studied since the sixties, when it became necessary to take into account the flexibility of the components in order to perform more accurate simulations and design lighter and more efficient structures. Beams, due to their importance in many engineering applications and their intrinsic simplicity, have received a particular attention. During the last four decades, several authors proposed different mathematical models with the aim of correctly estimating the modal characteristics and dynamic response of rotating flexible beams. Most studies come from researchers working in the aerospace field since rotating beams can be often used as a simple model for propellers, helicopter blades, turbine blades and satellite booms.

Owing to the huge amount of papers published on this topic, a comprehensive survey of the subject is beyond the scope of this paper. Some excellent reviews can be found in the related literature, which are extremely useful in identifying and tracking the major developments over a large span of time and can be also helpful in locating the contribution of the present

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work. In particular, the reader is referred to the review papers of Sharf [30] and Kunz [16], which cover most of the material up to the mid-nineties. An extensive overview of the most recent material can be found in the book of Hodges [10].

It is well known that, even though a flexible beam undergoes small elastic vibrations during rotation, several motion-induced stiffness terms can be missed in those formulations where linear strain quantities relative to a floating frame following the rotation of the beam are early adopted in the expression of the total potential energy of the system. This simplified scheme, which leads to linear models characterized by a spurious loss of bending rigidity, in the literature is often referred to as premature or inconsistent linearization of the deformation field during the derivation of the equations of motion [22].

One of the first attempts to account for motion-induced stiffening dates back to the paper of Houbolt and Brooks [12] in which a set of equations governing the coupled torsion-bending vibrations were derived for a twisted rotating beam. The linearization was retarded as much as possible to include all the first-order terms arising from the coupling between the tensile load induced by the centrifugal acceleration and the other strain measures.

Some years later, a similar approach was followed by Likins et al. [19], who derived the linear equations governing the out-of-plane transverse vibrations of spinning elastic bodies by accounting for the potential energy resulting from the centrifugal force. By neglecting the axial vibrations, they also discarded the gyroscopic coupling between the axial and bending motion due to the Coriolis acceleration, which instead is relevant as the angular velocity increases as shown by Yoo and Shin [36].

Vigneron [33] followed similar steps in deriving the linearized dynamic equations of rotating beams but he divided explicitly the axial displacement into one contribution due to the stretching of the beam elastic axis and another contribution related to the so-called foreshortening effect. In this way, the geometric stiffening effect appears in the kinetic energy associated with the foreshortening displacement component. This approach has strong similarities with the one proposed by Kane et al. [14], also known as "*foreshortening approach*", which is based on a hybrid set of deformation variables involving both non-Cartesian (stretch deformation) and classical Cartesian variables (transverse deformation). The use of the principle of virtual work combined with the stretch variable substitution yields a motion-induced stiffening contribution due to the work done by the inertial and external forces for that part of the virtual displacement related to foreshortening. When linearized equations are sought for, this method is equivalent to those classical formulations which include the elastic potential energy resulting from the centrifugal force [23,35,3].

A different approach was followed by Laskin et al. [18] and later by Lin and Hsiao [20] and Huang et al. [13] and Kim et al. [15], who decomposed the beam axial displacement into a small but finite quasi-static component, computed before solving the equations of motions, and an infinitesimal transient component. This method allows us to include the effect of the deformed geometry, which has a simple analytical solution for a beam rotating in a plane. However, as shown in the following, they do not foresee several stiffening terms, which should arise from the corresponding nonlinear term in the Green–Lagrange strain. This leads to the wrong prediction of a critical value of the angular velocity where the model exhibits an instability also for stiffening materials.

Another interesting study was presented by Banerjee and Dickens [2], in which a consistent linearization procedure was exploited to include several motion-induced stiffness terms for a generic deformable body in large overall motion. The work of the zeroth-order components of the stress tensor for the nonlinear part of the virtual variation of the Green–Lagrange tensor was retained in the formulation and allows us to capture some geometric stiffening components, although the authors did not provide the equations for the case of a rotating beam.

A more modern and rigorous approach of modeling rotating beams is based on the geometrically exact theory. The literature on geometrically exact beam theories (GEBTs) dates back to the seminal work of Reissner [28], where a large strain theory was first presented and later extended to the case of spatially curved beams and large displacements [29]. The kinematics of the geometrically exact beam was first introduced by Simo [31], but similar developments can be found in Borri and Merlini [4] and Cardona and Geradin [5]. In particular, the novelty of these works relies in the exact treatment of the orientation of the beam cross section. In Danielson and Hodges [6], the three-dimensional strain field has been written in terms of the intrinsic one-dimensional measures for initially twisted and curved beams. Hodges [9] has also derived a mixed variational formulation for the dynamics of moving beams which can be used once appropriate constitutive laws and kinematics relations are provided.

A solid and comprehensive theoretical background to the geometrically exact approach appeared about one decade ago in the books of Hodges [10] and Pai [24]. In the first monograph, the author presents the intrinsic formulation of the dynamic equations of initially curved and twisted beams. The cross-section analysis of the beam is also discussed extensively based on the variational asymptotic method [10]. The fully intrinsic approach, since it is devoid of displacement and rotation variables, is particularly attractive to overcome the issues associated with finite rotation variables. The modeling of highly flexible structures is the main subject of the book of Pai [24], where a general displacement-based formulation of geometrically exact beams, plates and shells accounting for shear deformations and warping effects is developed.

GEBTs are based on an exact description of the beam kinematics as a one-dimensional continuum and they rely on the definition of generalized strain measures, namely the *force* and *moment* strains. Most of these theories exploit Jaumann strains under the small strains and local rotation assumption, according to which second-order generalized strain components are neglected. The importance of these second-order terms in modeling rotating blades was highlighted by Masarati and Morandini [21] and Hodges [10], in particular for what concerns the torsional stiffening induced by a tensile load, the so-called trapeze effect. In a very recent paper, Pai [25] outlines several other problems and shortcomings present