Journal of Sound and Vibration 368 (2016) 135–147

Contents lists available at ScienceDirect

Journal of Sound and Vibration

journal homepage: www.elsevier.com/locate/jsvi

Modeling the dynamic stiffness of cracked reinforced concrete beams under low-amplitude vibration loads

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ARTICLE INFO

Article history:
Received 4 October 2015
Received in revised form
6 January 2016
Accepted 8 January 2016
Handling Editor: L.G. Tham
Available online 28 January 2016

Keywords:
Reinforced concrete
Crack
Vibration
Stiffness
Moment of inertia

ABSTRACT

In this paper, a model, initially developed to calculate the stiffness of cracked reinforced concrete beams under static loading, is used to assess the dynamic stiffness. The model allows calculating the average inertia of cracked beams by taking into account the effect of bending cracks (primary cracks) and steel–concrete bond damage (i.e. interfacial micro-cracks). Free and forced vibration experiments are used to assess the performance of the model. The respective influence of bending cracks and steel–concrete bond damage on both static and dynamic responses is analyzed. The comparison between experimental and simulated deflections confirms that the effects of both bending cracks and steel–concrete bond loss should be taken into account to assess reinforced concrete stiffness under service static loading. On the contrary, comparison of experimental and calculated dynamic responses reveals that localized steel–concrete bond damages do not influence significantly the dynamic stiffness and the fundamental frequency.

1. Introduction

Due to the low tensile strength of concrete, reinforced concrete structures in service are cracked. Concrete cracking reduces the stiffness of the structural members, which influences not only the static behavior but also the dynamic responses of the structures [1]. To assess the effect of cracks on the behavior of concrete structure, two types of effort have been made by scholars. The first one is the detection and identification of cracks in concrete structures, which is the so-called inverse problem [2–6]. Secondly, researchers focused on the static and dynamic behaviors of the cracked concrete structures (direct problem), which is the approach considered in this paper.

The key in researching the dynamic behavior of a cracked beams is to properly evaluate the stiffness or compliance of the damaged cross-sections [1]. Extensive experimental, analytical, and numerical investigations have been reported on the dynamic response of structures [7]. From an overall point of view, there are two basic methods adopted for this problem: nonlocal continuous cracked beam and local flexibility model.

The first approach assumes that the reduced stiffness of the cracked beams is nonlocal and continuously distributed along the span. Mirza et al. [8] presented static and dynamic tests on prestressed concrete box girders. The reduced flexural stiffness uniformly distributed along the span of the cracked girders was determined through static tests at different levels

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http://dx.doi.org/10.1016/j.jsv.2016.01.007
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of cracking damage. Based on dynamic experiments, Jerath and Shibani [9] developed a dynamic stiffness model, which is similar to the ACI-318-83 static stiffness model. Abdel Wahab et al. [10] used a damage function to describe the damage pattern of reinforced concrete beams based on three parameter: the length of the damaged zone, the magnitude of damage, and the variation shape of the elastic modulus. Similarly, Chondros et al. [11] and Bilello [12] considered that the stiffness of the beam can be reduced in a linear way.

In the second type of methods, using the fracture mechanics theory, the stiffness of cracked beam was modeled as discrete springs between two adjacent segments. As a result, quantifying the equivalent massless torsional spring models is the main challenge in this approach. Zhao et al. [1] reviewed five popular models [13–17] and expressed them as a uniform equation with dimensionless constant local compliance. Neild et al. [18] developed a discrete model by virtue of a time-stepping method, where the stiffness of the spring can vary with time.

Previous models of vibrating cracked beams used either open crack model or breathing crack models [19]. Breathing crack is a crucial phenomenon [20,21], where the crack is assumed to be either open or closed, leading to a step change in stiffness. However, in the reinforced concrete the crack problem becomes more complicated. Neild et al. [22] have shown experimentally that once a crack has been opened during loading, loose aggregate may prevent the crack from closing when unloading and that the interaction between the steel and the concrete should be taken into account.

A model allowing and calculating the average moment of inertia of cracked RC-beams was developed by Castel et al. [23–25] based on a linear bond relationship between steel and concrete between two concrete cracks. The overall beam stiffness under static cyclic loading was derived from that assumption. The model was successfully verified experimentally. In this model, two types of crack namely primary and interfacial microcracks are considered. The tension stiffening between the primary cracks is acknowledged in the model [23,24]. Meanwhile, the reduction in tension stiffening due to interfacial microcracks (cover-control cracking) was considered as well via a damage variable D_{ccc} modeling the loss of bond [25,26].

In this paper, the Castel model [23–25] is used to analyze the dynamic response of RC-beams. The influence of both primary cracks and interfacial microcracks is assessed. Static loading, free vibration, and low-amplitude forced loading experiments from the literature [27–29] are used to assess the performance of the model to predict both static and dynamic response of the beams. The moments of inertia are calculated, taking into account the effects of primary cracks only or the combined effects of primary cracks and interfacial microcracks, in order to evaluate the influence of the steel–concrete bond loss versus the influence of the primary cracks.

2. Modeling moment of inertia of cracked reinforced concrete

As shown in Fig. 1, the path “OABC” is the monotonic load-deflection envelop obtained by performing a static loading test beyond cracking loading P_{cr}. The stiffness of the reinforced concrete beam can be described by using the effective moment of inertia (I_e) model [30,31]. Owing to the cracking damage, when unloading, the beam follows the path “BD”. The deflection does not come back to zero, leading to an irreversible deflection (“OD”). The stiffness of the beam subjected to loading cycle following the path “BD” is governed by the average moment of inertia labeled I_a as evidenced by Ref. [24]

From the perspective of cracking formation during loading process, after the cracking load has been reached, a further increase in loading leads to the formation of cracks labeled primary bending cracks, of which widths are millimeter-sized as plotted in Fig. 2. When the steel reinforcement stress at crack location reaches a threshold, the interfacial microcracks’ formation in the concrete located between the primary cracks is triggered [26,32]. These interfacial microcracks are micron-sized as shown in Fig. 2.

In this section, the effects of both primary and interfacial microcracks are considered in the calculation of the average moment of inertia I_a of cracked reinforced concrete beams.

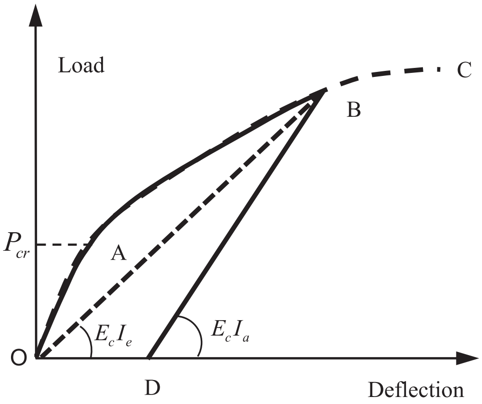

Fig. 1. Typical overall response of RC-beams during a loading cycle.